

A
COMPENDIUM
OF
Algebra.

CONSISTING OF
*Plain, Easie and Concise Rules for the
Speedy attaining to that ART.*

Exemplified by various *PROBLEMS*,
with the *Solution* of their *Æquations* in *Numbers*.

BY

A *New and General Method* of *Resolving* all kind
of *Æquations*, both *Pure* and *Adfected*, with
great *Ease* and *Expedition*; very different
from all others yet *Extant*.

APPLIED

To *Squaring* the *Circle*, *Making* of *Sines*, *Tan-*
gents, *Secants*, and *Logarithms* with great *facility*.

ALSO

An *Appendix* concerning *Compound-Interest* and *Annuities*.

By *JOHN WARD*, *Phylomath*.

Heretofore *General Gauger* in the *Revenue* of *Excise*.

L O N D O N :

Printed for the *Author*, and by him Sold at the *Black-Boy
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Church* in *Fleet-street*; and in *Threadneedle-street*. 1695.

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TO THE
Honour'd, and Learned
Mr. JOSEPH RAPHSON,
Fellow of the Royal Society.

SIR,

THE *Mathematical Sciences* have always been found of such general Use and Advantage amongst Mankind, That 'tis no wonder all Ages have produc'd so many Great and Eminent Persons, who have employ'd their Studies for the Improvement of those Excellent Arts, wherein their Labours have had great Success; such were *Euclid*, *Archimedes*, *Ptolomy*, *Galilaus*, &c. But to omit all others, and come to our own Country and Age, What great Improvements have been lately made? To instance only that of *Logarithms* (the Noble Invention of the Noble Lord *Napeir*) so useful and necessary in the most considerable Parts of *Mathematical Operations*. Not inferiour to any of these, is your own Discovery of Resolving *Adfected Equations*, the Value and Advantages of which, succeeding Ages will be enforc'd to acknowledge, as

The Epistle Dedicatory.

that which has not only awaken'd, but encourag'd the Sublimest of *Mathematical Genius's* to proceed in the further Discoveries of such Mysteries as lie yet conceal'd, and by the more Supine have been thought Impossibilities. Among the rest, I (although the Meanest, yet not the least Laborious) have presum'd to give the World a Taste of the Effects of some Years Experience. And these my first Fruits I humbly Present to your self (worthy Sir) as being a Person, who for your peculiar Learning, Pains and Industry bestowed on this particular Study, may deservedly be deem'd the most Competent and Proper Judge of these ensuing Pages, which if you are pleas'd to Accept and Approve, you will thereby become my Patron to protect me from the Censures of others, I am

S I R,

Your most Obliged,

And Humble Servant,

JOHN WARD.

TO THE
R E A D E R.

I Have adventur'd in this Critical Age, to present the World with these few Sheets, which are a Compendium of (that Universal Part of the Mathematics) Algebra; not insensible, that several very Eminently Skilful in all Parts of Mathematical Literature, have both Largely and Learnedly handled this most difficult Subject: The Thoughts of which almost prevented my Attempting any thing of this Nature. But when I consider'd that most of those Authors Works were so voluminous, that many Persons who have a Mathematical Genius or Fancy tending that way, might perhaps want Time, or other Conveniences suitable to the Perusal of such Authors; and not only so, but several very well versed in other parts of those Studies, have wholly declined this Art, being deterr'd from it by the Difficulty and Tediousness of Resolving Affected \AA equations into Numbers, there never having been in any Book that I have had the opportunity of perusing, (Mr. Raphson's *Analysis* *\text{\AA}equationum Universalis* excepted) any easie Method prescribed for effecting the same; which I was most certain I could perform with great Facility and Expedition.

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To the Reader.

These Considerations prevail'd with me to believe, That if there were such a Treatise as began with the first Rudiments of Algebra, and proceeded gradually through the several Parts of it, plainly shewing how to form, and bring any Problem to an Equation, and then shew how to Resolve that Equation into Numbers, it would undoubtedly be very Acceptable to all Lovers of Art.

For these Reasons I compos'd this Compendium, contracting the same (both for Convenience of Price and Portability) into as narrow a Compass as possibly I could; (though the Contents of it would have filled a Larger Book,) and yet I presume there is not any thing omitted that might conduce towards the Learners Attaining to a perfect Knowledge of this Mystrious Art, which is indeed the very Principal or Kernel of all the Mathematical Sciences, as being the Inventive Part.

To be brief, the whole is Methodical, Plain and Easie, the Rules being digested and freed from all Obscurity, and explain'd by various Examples in each part of it, beginning with (what I call) Algebraic Arithmetic, treating of every Part distinctly.

Next, I introduce the Learner into the use of the same, by Plain and Easie Questions in Numbers, thereby teaching him how to bring Quantities to an Equation, &c. Then I shew how to
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To the Reader.

Extract the Roots of all pure Æquations, viz. Of the Square, Cube, Biquadrate, Surfolide, &c. to any Height or Power whatsoever; by a New and Easie Method (never before Published.)

After that, I Proceed to the Resolving of Geometrical Problems, fitted for the raising Various Forms of Adfectèd Æquations, Resolving each into Numbers (shewing the Construction, or Rise of each Theorem for their Solution) as they are Produced, Applying the same to a speedy Approximation for Squaring the Circle, Making of Sines, Tangents, and Secants, with a New and Speedy Method of making Logarithms, &c.

I have also annexed (though something forreign to this Subject) a Short Appendix of Compound Interest and Annuities, shewing the Foundation, or Rise of those Rules, by which Questions of that Nature are performed.

These, with several other Compendious Contractions not mention'd here nor in the Title, are to be found in this small Treatise. I am not sensible of any Deficiency therein; only, the rough and unpolish'd Style in which it is deliver'd, may perhaps leave some room for the Cavils of Critics, more especially those that seek for Opportunities; for such I know there are, that will pick any Quarrels, when out-done, or contradicted in their common Practice: However, not to such, but to the Ingenious, I willingly submit, not doubting of their candid

To the Reader.

candid and favourable Acceptance, hoping withal, they will receive ample Satisfaction from these my Labours, which is the only thing desir'd, and aim'd at, By

April 10th,
1695.

Their Real,

And Hearty Well-wisher,

JOHN WARD.

ERRATA.

NOTwithstanding great Care hath been taken for the avoiding Errors of the Press (which Books of this kind are very subject to) some Literal Faults have escap'd, such as in the First Page, *Definition* for *Definition*, and *Obtruce* for *abstruse*, with some few others of the like kind elsewhere, of which, these are the most material, *viz.*

Page 15; Line 27, for *Second Example*, read *First Example*.

Page 25, Line 21, for *Compassion*, read *Composition*.

Page 79, Line 18, for $c + \frac{3RRa - aaa}{2RR}$
read $:\frac{1}{2}c + \frac{3RRa - aaa}{2RR}$

Page 85, Line 15, for *Call us*, read *tell us*.

Page 87, Line 26, from the same, read for the same.

Besides these, I dare not warrant that none have escap'd unseen; but if there happen to be any, I am very sure they are such as can no way discourage, or hinder a Learner from tracing each particular Process, both in the Specious and Arithmetical Operations.

A

A

COMPENDIUM OF ALGEBRA.

CHAP. I.

Of the Nature of Algebra, and Definition of the Symbols, or Characters used therein, &c.

I Shall not undertake (in this small Tract) to give an Historical Account of the Original, or Antiquity either of this Art, or its Name; but confine my self within the Limits of this Definition, that (that which is now called) *Algebra*, is an Art by (*the help of*) which the most difficult and obtruse Questions, either in *Arithmetick*, or *Geometry*, may be Resolved, by an Universal Method of Reasoning, and comparing of known Quantities with those that are unknown.

B

And

Algebraick Definitions.

And this way of arguing, or comparing of Quantities with Quantities, not only discovers an Answer to the Question proposed, but at the same time doth raise (and demonstrate) Theorems, or Rules, for the Resolving of all Questions in the like Nature.

And from hence it is, that so many Curious and Useful Theorems, Canons, or Rules are first discovered, which are of excellent use in the Practice, but would for ever lie hid were it not for this Art.

This *Algebraick Arithmetick*, is performed by substituting of Alphabetical Letters, to represent as well the known, as the unknown Quantities concern'd in any Problem, (be it Arithmetical, or Geometrical.) And to these Substituted Letters, are prefixed certain Symbols, Notes or Characters, by the help of which, the Quantities (in their substitutes) are so connected, and ordered, that thereby they become capable of being compared, and moulded by Addition, Subtraction, Multiplication, and Division, &c. according to the Design or Nature of the Question: And this is so done, as tho' all the Quantities concern'd therein, (as well the sought as the given) were really known; till at last the given Quantities comes to an Equation (or Equality) with the Quantity that's sought, (or some Power thereof) that so it may be found.

In all which various Operations and Comparisons, all the Quantities concern'd in the Question, remain in their Species so plain and clear, that they neither burden the Memory, nor rack the Fancy; but on the contrary help both, by representing to the Eye (at once) the Process of each Operation, used throughout the whole Management of a Question.

The Method of noting down these Substitutes in this Tract is thus. I make choice of representing the Quantity sought (be it a Line, or Number) by the small (*a*), and if more than one are sought, I make use of the Vowels (*e*), or (*y*), &c. to represent them. And for the given Quantities,

Algebraick Definitions. 3

ties, (be they either Numbers, Lines, or other Magnitudes,) I represent them by the Consonants *b, c, d, f, g, &c.* And for Distinction I note (as usual) all Schemes with the Great or Capital Letters, and work all Operations with the small Letters.

If any Quantity be taken more than once, prefix its Number; as $6a, 5b, 9c, 3\frac{1}{2}d, &c.$ these stand for a , taken Six times, or Six times a , Five times b , &c. And all Numbers thus prefixed to any Quantity, are called Coefficients (or fellow Factors) but if the Quantity have no prefixed Number, (or Coefficient) then such Quantity stands for itself once taken, and is supposed (or understood) to have an Unit prefixed to it, as a , is $1a$, or b , is $1b, &c.$

The Signs made use of in this Tract, for the more short and quick expressing of Words, are the most General now in use, and are those following.

N. B. Note, That it is very requisite for the Young Learner of this Art, to be very perfect in the knowledge of the Signs, and their Significations, before he proceeds further therein; that is, before he intermedles with any of the following Rules.

Also I advise him to be very ready in one Rule, before he undertakes the next; that is, to be expert in, and understand *Addition*, before he medles with *Substraction*; and that of *Substraction*, before he undertakes that of *Multiplication*, &c. because they have each a Dependency one upon the other.

And by this method of proceeding, he may make himself Master thereof, or at least arrive to a competent knowledge therein, with ease, and in a very short space of time.

Algebraick Definitions.

<i>Signs.</i>	<i>Names.</i>	<i>Significations.</i>
$+$ $\}\}$	<i>More.</i>	The Affirmative Sign, and Sign of Addition, as $a + b$, signifieth that the Quantities, a , and b , are to be added together.
$-$ $\}\}$	<i>Less.</i>	The Negative Sign, and Sign of Subtraction, as $a - b$, signifieth that the Quantity b , is to be taken from that of a .
\times $\}\}$	<i>Into.</i>	The Sign of Multiplication, as $a \times b$, signifieth that a , is to be Multiplied into b , but oftentimes the Sign is omitted, yet is understood, as ab , is $a \times b$.
\div $\}\}$	<i>By.</i>	The Sign of Division, as $a \div b$, signifieth that a , is to be Divided by b ; yet this Sign is also often omitted, as $\frac{a}{b}$ is $a \div b$, or thus; $b) a ($
$::$ $\}\}$	<i>Proportion.</i>	The Sign of Disjunct Proportion, as in the <i>Golden Rule</i> , and is thus, $a : b :: c : d$. to be read in this manner, as a , is to b , so is c , to d .
\div $\}\}$	<i>Continued.</i>	The Sign of Continued, or Geometrical Proportion; that is, as $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa}$ &c.
\therefore $\}\}$	<i>Ergo.</i>	Signifying the Product of the Two Extreames, is equal to that of the Means, and $[\therefore]$ is placed in the Margin after Proportion.

Signs.

Algebraick Definitions.

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Signs.	Names.	Significations.	
$= \} \}$	<i>Equal.</i>	The Sign of <i>Æquation</i> , or <i>Equal</i> to, as $a = b$, signifieth that a is equal to b , &c.	
$\odot \} \}$	<i>Involve.</i>	The Sign of <i>Involution</i> , and is placed in the Margin of a Process, to signify, that such Quantity, or <i>Æquation</i> to which it refers, is to be Multiplied by it self, &c.	
$\omega \} \}$	<i>Evolue.</i>	The Sign of <i>Evolution</i> , and is placed in the Margin, to signify the contrary to <i>Involution</i> , viz. The Extraction of the Root, &c.	
$\sqrt{\} \} \}$	<i>Root.</i>	The Sign of <i>Irrationallity</i> , or of a <i>Surd Root</i> ; over which, if a Number be placed, it signifieth of what degree that Root is: And this Number is called the <i>Index</i> , thus	
$\sqrt[2]{\} \} \}$	Denotes the	<i>Square</i>	} Root, &c.
$\sqrt[3]{\} \} \}$		<i>Cube</i>	
$\sqrt[4]{\} \} \}$		<i>Biquadrate</i>	
$\sqrt[5]{\} \} \}$		<i>Surfolide</i>	
$\sqrt{\}$	Signifieth an <i>Universal Root</i> , or <i>Root of Roots</i> .		
$> \} \}$	<i>Greater.</i>	The Sign of <i>Majority</i> , as $a > b$, signifieth that a is greater than b .	
$< \} \}$	<i>Lesser.</i>	The Sign of <i>Majority</i> reversed, as $b < c$, signifieth, that b , is <i>Lesser</i> than c .	
$\square \} \}$	<i>Square.</i>	Signifieth the <i>Square</i> of any Numbers, or Quantities, or of an <i>Æquation</i> , as $\square a + b + c$, &c. is the <i>Square</i> of a , more b , more c .	

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B 3

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6 Algebraick Definitions.

The chief of these Signs, or Characters wherewith the Substituted Quantities (or Letters) are connected, be More, and Less, *viz.* The Sign $+$, and Sign $-$; where *Note*, That the Sign always belongs to the Quantity that follows it, (or is to the Right Hand thereof,) and not to that which is before it. And if a Quantity hath no Sign prefix'd before it, (as generally the Leading Quantity hath not) it's then understood to have the Sign $+$ prefixed, as a , is $+$ a , or $b + d$, is $+$ $b + d$, &c.

If several Quantities be connected, or engaged in any Question, they may stand in any order at pleasure; as $a + b - d$, may stand thus, $a - d + b$; or thus, $-d + a + b$, &c. being still the same, tho different-ly placed; and these are called Compound Quantities, that is, when Two or more different single Quantities are connected together by their Signs, of which more in its due place.

The Method used in this Tract, of placing Figures, and Signs in the Margin, according to the several Steps, or Process used in forming of an Equation, is the same with that of Mr. *Brannkers*, (or rather Dr. *Pells*,) in his *Introduction to Algebra*, Printed 1668, (which hath of late been followed by others) and is thus to be understood.

The Numbers 1, 2, 3, 4, &c. in the small Column, do shew the several Steps used in the forming, or bringing the Quantities concerned in any Question to an Equation: And the Numbers placed in the Margin, have relation to those in the small Column, and the Signs placed betwixt the said Numbers in the Margin, do shew, how the Quantities that stand against the like Numbers in the small Column have been ordered, either by Addition, Subtraction. or otherwise. And this Method of Registering the several Processes, is very useful, especially in long and tedious Operations, as will more fully appear in the Practice thereof. An Example or Two will help to explain the same better than Words.

Suppose

Algebraick Definitions.

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Suppose it were required to set down the Sum of the Two Quantities, a , and b , according to this Method.

Thus

1	a
2	b
1 + 2	$a + b$

Having placed down the Quantities a and b , against the Figures 1, 2, in the small Column, also against 3, must be placed 1 + 2 in the Margin, which denotes that the Quantities in the First and Second Steps, are added together, and those in the Third Step, are their Sum.

To illustrate this in Numbers, suppose $a = 8$, and $b = 6$.

	Quantities.	Numbers.
1	a	8
2	b	6
1 + 2	$a + b$	$8 + 6 = 14$

Again, suppose it were required to set down the Difference of the same Two Quantities.

Thus

1	a	8
2	b	6
1 - 2	$a - b$	$8 - 6 = 2$

Again to set down their Product.

Thus

1	a	8
2	b	6
1 x 2	ab	$8 \times 6 = 48$

These Definitions of the Signs, (or Notes) Method, &c. being premised, and pretty well understood by the young Learner, (for so they ought to be,) He may then proceed to the several parts of *Algebraick Arithmetick*, viz. *Addition*, *Subtraction*, *Multiplication*, and *Division*, &c.

In the delivery of which, I shall begin with single whole Quantities, in each distinct part thereof, then proceed in the like Order with Compound Quantities, and so on to Fractions, treating of each separately, with what plainness and brevity I can.

CHAP.

C H A P. II.

Of the several parts of Algebraick Arithmetick, and how the same are performed.

§. 1. Addition of Single whole Quantities.

Rule 1. IF the Quantities be a like, and have like Signs, add the Coefficients (or prefixed Numbers) and adjoyn the Quantities with the same Sign.

Thus	1	a	$- 3b$	$+ 5bc$	$- 6abc$
	2	a	$- 5b$	$+ 4bc$	$- 7abc$
$1 + 2$	3	$2a$	$- 8b$	$9bc$	$- 13abc$

Rule 2. When the Quantities be alike, but have unlike Signs, subtract the Coefficient from each other, and adjoyn the Quantities with the Sign of the Greater.

Thus	1	$3a$	$- 5b$	$- 9bd$	$10abcd$
	2	$- a$	$+ 7b$	$+ 7bd$	$- 12abcd$
$1 + 2$	3	$2a$	$+ 2b$	$- 2bd$	$- 2abcd$

Rule 3. But when the Quantities be unlike, (be the Signs alike or unlike) set down the Quantities with their respective Coefficients, without altering their Signs, and thence ariseth Compound Quantities:

Thus	1	a	a	$- 7bcd$
	2	b	$- b$	$+ 5bdg$
$1 + 2$	3	$a + b$	$a - b$	$- 7bcd + 5bdg$

Also thus	1	a		$- 5a$
	2	b		$+ 7bc$
	3	c		$- 40$
$1 + 2 + 3$	4	$a + b + c$		$7bc - 5a - 40$

§. 2. Sub

§. 2. Subtraction of Single whole Quantities.

General Rule.

CChange, (or at least in your Mind suppose to be changed) all the Signs of the Subtrahend; then add, or collect all the Quantities together, (as before in Addition) and that Sum will be the Remainder, or Difference sought.

For Example, Suppose it were required to find the Difference of these Quantities.

$$\begin{array}{r|l|l|l|l} 2a & 8b & -7bc & 2ba \\ a & 5b & -4bc & 2ba \end{array}$$

According to the Rule, the same Quantities will stand, when their Signs are changed thus.

$$\begin{array}{r|l|l|l|l} 1 & 2a & -8b & -7bc & 2ba \\ 2 & -a & -5b & +4bc & -2ba \\ 1-2 & 3 & 3b & -3bc & 0 \end{array}$$

But this may be done by supposing the Signs changed, without setting them down again.

$$\begin{array}{r|l|l|l|l} \text{Thus} & 1 & 5a & -4bc & -5bd & 0 \\ & 2 & -2a & +5bc & -8bd & -2ba \\ 1-2 & 3 & 7a & -9bc & +3bd & +2ba \end{array}$$

$$\begin{array}{r|l|l|l|l} \text{Again,} & 1 & a & -4b & +5bcd \\ & 2 & b & -3a & -7baf \\ 1-2 & 3 & a-b & +3a-4b & 5bcd + 7baf \end{array}$$

Thus Compound Quantities, are produced by *Subtraction*, as they were by *Addition*.

Note, That *Subtraction* is proved, by adding the Subtrahend to the remainder; as in vulgar Arithmetick.

$$\begin{array}{r|l|l|l|l} \text{Thus} & 1 & 7b & -5bc & -4abd \\ & 2 & -2b & +3bc & -3cfa \\ 1-2 & 3 & 9b & -8bc & +3cfa - 4abd \\ 2+3 & 4 & 7b & -5bc & -4abd \end{array}$$

§. 3. Mul-

10 Algebraick Arithmetick.

§. 3. Multiplication of single whole Quantities.

Rule 1. IF the Quantities have no Coefficients, and have like Signs, joyn them together, whether they be like or unlike, and to the Product prefix the Sign +.

Thus	1	a	ab	$-bc$	$-ab$
	2	b	cd	$-df$	$-d$
1×2	3	ab	$abcd$	$+bcd$	abd

Rule 2. If there be Coefficients, Multiply them, and to their Product joyn the Quantities, (set together as before.)

Thus	1	$5a$	$7bc$	$-24bc$	$-ab$
	2	b	$4ad$	$-6df$	$-d$
1×2	3	$5ab$	$28adbc$	$+144bcd$	abd

Rule 3. And when the Quantities have unlike Signs, the Operations are the same with the foregoing; only to the Product prefix the Sign -.

Thus	1	a	$-bc$	$-7b$	$-ba$
	2	$-b$	$+cd$	$+6ad$	$+12$
1×2	3	ab	$-bcd$	$-42bad$	$-12ba$

From hence it appears, that like Signs produce the Affirmative Sign +, but unlike produce the Negative Sign -.

The Reason thereof may be thus stated: Suppose $a =$ any Affirmative Quantity, and let $-b = -2$, then is b equal to 2 less than nothing; (for so are all Negative Quantities.) Now to Multiply any Number or Quantity, implies a putting together of the same, so often as is the Number in the Multiplier, as $a \times 2 = 2a$, or a , twice put together. But if the Multiplier be -2 , then it implies the taking away of a , twice; and therefore the Product must needs be $-2a$, that is, $a \times -2 = -2a$, or $a \times -b = -ab$, as above. But to Multiply $-a$, into -2 , by the same Reason, it will take away the Defect of a , twice, which is the same as to supply it; therefore $-a \times -2 = +2a$, or $-a \times -b = ab$.

§. 4. Di-

Algebraick Arithmetick.

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§. 4. Division of single whole Quantities.

Rule 1. IF they have like Signs, and no Coefficients, and if there be like Quantities in the Dividend, as there is in the Divisor, cast off such Quantities in both, and set down the other Quantities, with the Sign +.

Thus

1	ab	$abcd$	$- bcf$	$- abc$
2	a	bc	$- bf$	$- abc$
3	b	ad	$+ c$	$+ 1$

$1 \div 2$

N. B. Note, That when the Quantities in the Dividend and Divisor be all the same, then the Quotient is an Unity, as in the last above.

Rule 2. But if the Quantities have Coefficients, divide the Numbers (as in common Arithmetick,) and to the Quotient prefix the Quantities.

Thus

1	$6 ab$	$- 12 abc$	$18 cd$	$- 8 adf$
2	$3 a$	$- 3 ab$	$3 c$	$- 2 adf$
3	$2b$	$+ 4 c$	$6 d$	$+ 4$

$1 \div 2$

Rule 3. When the Quantities have unlike Signs, the Operations are the same; only set down the Quotient with the Sign —.

Thus

1	$- ab$	$+ 6 ab$	$+ 12 cd$	$- 8 ad$
2	$+ a$	$+ 3 a$	$- 4 c$	$+ 4 ad$
3	$- b$	$+ 2 b$	$- 3 c$	$- 2 ad$

$1 \div 2$

$\begin{matrix} - 2b \\ - 2 \end{matrix}$

But when the Quantities in the Divisor, are not found in the Dividend, set down both ~~both~~ Fraction wise ~~the~~.

Thus

1	a	$6 bc$	bdf	abc
2	b	$3 d$	abf	$4 ac$
3	$\frac{a}{b}$	$\frac{2 bc}{d}$	$\frac{d}{a}$	$\frac{b}{4}$

$1 \div 2$

§. 5. In

§. 5. Involution of single whole Quantities.

Involution is performed as *Multiplication* is, (and indeed is no more but *Multiplication* differing only in this, *Multiplication* may be perform'd by different Multiplicators, but *Involution* still retains the same.) If any Quantity (or Quantities) be involved, or drawn into itself, and afterwards into that Product; or again, a third time, into the last Product, &c. as manifold as is the power, so great must the Number be that is used to express it; and this Number is called the Index of the power, and is set after the Sign in the Margin.

Thus	1	a	$- 2a$	bc
1 @ 2	2	aa	$+ 4aa$	$bbcc$
1 @ 3	3	aaa	$- 8aaa$	$bbbccc$
1 @ 4	4	$aaaa$	$+ 16aaaa$	$bbbbcccc$
1 @ 5	5	$aaaaa$	$- 32aaaaa$	$bbbbbbcccc$

Which may be shortened, by placing of the Indices over the Quantity (or Quantities) according to the height of the Power; by such Method the Work will stand thus.

	1	a	$- a$	bc	Root
1 @ 2	2	a^2	$+ a^2$	$b^2 c^2$	Square
1 @ 3	3	a^3	$- a^3$	$b^3 c^3$	Cube
1 @ 4	4	a^4	$+ a^4$	$b^4 c^4$	Biquadraticke
1 @ 5	5	a^5	$- a^5$	$b^5 c^5$	Surfolide, &c.

§. 6. Evo

§. 6. Evolution, (or Extracting of Roots) of single whole Quantities.

Rule. IF the Quantity have a Root, (which is easily discover'd, by comparing its Index with the Root required,) divide the Index of its Power, by 2, if the Square Root be desired; or by 3, if you require the Cube Root, &c. and the Quotient will give the Root required, dealing with the Coefficients, or prefixed Numbers, as in Vulgar Arithmetick.

Thus	1	a^2	36 bb	81 $bbcc$
1 w 2	2	a	6 b	9 bc

	1	a^6	8 bbb	$bbbddd$
1 w 2	2	a^3	$\sqrt{8} bbb$	\sqrt{bbbddd}
1 w 3	2	a^2	2 b	bd

But if either out of the Indices, or out of the Coefficients, no exact Root can be extracted, according to the Root required; then prefix before it the Sign of the required Root; and thence will arise a *Surd Quantity*, as in the Square Root of the Two last Examples above; and such are some of these following.

	1	a^6	— 32 a^5	ab	216 a^3
1 w 2	2	a^3	$+\sqrt[3]{32} a^5$	$\sqrt[3]{ab}$	$\sqrt[3]{216} a^3$
1 w 3	3	a^2	— $\sqrt[3]{32} a^5$	$\sqrt[3]{ab}$	6 d
1 w 4	4	$\sqrt[3]{a^5}$	$+\sqrt[3]{32} a^5$	$\sqrt[3]{ab}$	$\sqrt[3]{216} a^3$
1 w 5	5	$\sqrt[3]{a^5}$	— 2 a	$\sqrt[3]{ab}$	$\sqrt[3]{216} a^3$

Thus I have gon through the Six parts of *Algebraick Arithmetick*, in single whole Quantities, which being well considered, (and rightly understood) the following will be very easie; *viz.* Compound Quantities.

CHAP. III.

The parts of Algebraick Arithmetick
in Compound Quantities.

§. 1. **A**ddition of Compound Quantities is perform'd in the same manner, and by the same Rules as single Quantities are; viz. by putting together like Quantities, respect being had to the Signs; and in case the Quantities be not alike, connect them together by their Signs, as in the last case of Single Quantities.

Thus

1	$a + e$	$a + e$	$+ 3a - b + 3c$
2	$a + e$	$a - e$	$- 3a + b - 2c$
1 + 2	3	$2a + 2e$	$2a$
			c

Again,

1	$a - b + c - 7$	$- 4bc + bcd$
2	$a - b - c + 3$	$+ 2bc + 3bcd$
3	$- a + 2b + d + 6$	$+ 4bc - 2bcd$
1 + 2 + 2	4	$a + d + 2$
		$2bc + 2bcd$

Again,

1	$aa + 2ab + bb$	$a^3 + 3bba + bbb$
2	$- 4ab$	$- 6bba$
1 + 2	3	$aa - 2ab + bb$
		$a^3 + 3bba + bbb$

§. 2. *Subtraction* in Compound Quantities, is in all respects like that of Single Quantities, care being taken of their Signs as before.

Thus

1	$a + b$	$2a + b + 2c$	$4a + 3b - 7c$
2	$a - b$	$a + b + c$	$- a + 4b + 3c$
1 - 2	3	$2b$	$5a - b - 10c$

Again,

1	$aa + 2ab + bb$	$3a - b - 3c$
2	$+ 4ab$	
1 - 2	3	$aa - 2ab + bb$
		$- 3a + b + 4c$

§. 3. *Mul.*

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§ 3. *Multiplication* of Compound Quantities is the same with that of Single Quantities; for the Product of each Member of the Multiplier, into all the Members of the Multiplicand, (respect had to the Signs) is the Product.

$$\begin{array}{r|l|l} \text{Thus} & \begin{array}{l} 1 \\ 2 \end{array} & \begin{array}{l} 3a + 5d \\ b \end{array} & \begin{array}{l} 2b + c - 4d + f \\ 2bc \end{array} \\ 1 \times 2 & 3 & 3ab + 5db & 4bbc + 2bcc - 8cbd + 2bcf \end{array}$$

Again at large,

$$\begin{array}{r|l|l} & \begin{array}{l} 1 \\ 2 \end{array} & \begin{array}{l} a - b + c \\ b - d \end{array} \\ 1 \times b & 3 & ab - bb + bc & \text{Product by } b \\ 1 \times d & 4 & -ad + bd - dc & \text{Product by } -d \\ 1 \times 2 & 5 & ab - bb + bc - ad + bd - dc. \end{array}$$

Again,

$$\begin{array}{r|l|l} & \begin{array}{l} 1 \\ 2 \end{array} & \begin{array}{l} a + d + f - 3 \\ b - c \end{array} \\ & & \begin{array}{l} ab + db + fb - 3b \\ -ac - dc - fc \end{array} & \begin{array}{l} \text{Product by } +b \\ 3c, \text{ Product by } -c \end{array} \\ 1 \times 2 & 3 & ab + db + fb - 3b - ac - dc - fc - 3c \end{array}$$

§ 4. *Division* of Compound Quantities, is perform'd like that of Single; and if any Doubts should arise therein, they are easily reconciled, by considering, That *Division* dissolveth what *Multiplication* puts together; and therefore to be perform'd by contrary Operations.

$$\begin{array}{r|l|l} \text{Thus} & \begin{array}{l} 1 \\ 2 \end{array} & \begin{array}{l} 3ab + 5db \\ b \end{array} & \begin{array}{l} 2bcf + 4bbc + 2bcc - 8bcd \\ 2bc \end{array} \\ 1 \div 2 & 3 & 3a + 5d & f + 2b + c - 4d \end{array}$$

Thus the Product of the Second Example in *Multiplication*, divided by $2bc$, gives $2b + c - 4d + f$.

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Examples of Division at large.

Suppose the Product of the last Example in *Multiplication*, were given to be divided by $b - c$, the Operation will be thus perform'd.

	1	$ab - ac + db - dc + bf - fc - 3b + 3c$	
	2	$b - c$	
$2 \times a$	3	$ab - ac$	$(+ a$
$1 - 3$	4	$+ db - dc$	$(+ d$
$2 \times d$	5	$+ db - dc$	
$1 - 5$	6	$+ bf - fc$	$(+ f$
$2 \times f$	7	$+ bf - fc$	
$1 - 7$	8	$- 3b + 3c$	$(- 3$
2×3	9	$- 3b + 3c$	
$1 - 9$	10		0
$1 \div 2$	11	$a + d + f - 3.$	Each Quotient Collected.

Oftentimes the whole Process is omitted, only set down with the Sign \div . Thus $aa - bb \div a$, or $b + a$) $aa - bb$

When the Dividend will not be just wrought off without a Remainder, in such Cases, set all down (like the terms of a Vulgar Fraction,) as in the last Example of Single Quantities.

Thus	1	$ab + cb$	$aa + 2ab + cc$
	2	$d + f$	$a + 2b$
$1 \div 2$	3	$\frac{ab + cb}{d + f}$	$a + \frac{cc}{a + 2b}$

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§. 5. *Involvement* of Compound whole Quantities, is performed in all respects like *Multiplication*.

Example. Suppose the Square, Cube, or any other Power were required, from the Compound Root $a + b$, or $a - b$, viz. a Binomial or Residual Root.

		Binomial.	Residual.
	1	$a + b$	$a - b$
		$a + b$	$a - b$
1 × a	2	$aa + ab$	$aa - ab$
1 × b	3	$+ ab + bb$	$- ab + bb$
1 ⊙ 2	4	$aa + 2ab + bb$	$aa - 2ab + bb$
		$a + b$	$a - b$
4 × a	5	$a^3 + 2aab + abb$	$a^3 - 2aab + abb$
4 × b	6	$+ aab + 2abb + b^3$	$- aab + 2abb - b^3$
1 ⊙ 3	7	$a^3 + 3aab + 3abb + b^3$	$a^3 - 3aab + 3abb - b^3$

Thus are the Square and Cube produced from their Roots; and by this Method of Multiplying by the Root, may any Power be raised from any Compound Quantities given: Other Methods there are, but this I take to be the plainest, (and most Natural) for a Learner's Practice.

Again, Suppose the Square of $a + b + c$, was required.

Trinomial Root.

	1	$a + b + c$
		$a + b + c$
1 × a	2	$aa + ab + ac$
1 × b	3	$+ ab + bb + bc$
1 × c	4	$+ ac + bc + cc$
1 ⊙ 2	5	$aa + 2ab + bb + 2ac + 2bc + cc$

The Use, and easier Construction of Powers arising from 2, 3, 4, or more Members in the Root, shall be shew'd hereafter.

§ 6. *Evolution* of Compound Quantities, is the Converse of *Involution*, (and consequently perform'd by contrary Operations.) Whoever intends to be very dexterous at *Evolution*, or extracting of Roots out of Compound Quantities, remaining in their Species, must be very ready, and understand well the Genifis, or raising of Powers from their Roots; otherwise it will be difficult to discover whether the Given Quantities have a true Root or not.

The Problems in this Tract require no such Operations; yet that it may not appear deficient, observe this easie Rule.

Take so many distinct Quantities, as there are concerned in the Given Power to be evolved, and involve those Quantities (so taken) to the height of the Given Power, (without respect to the Signs;) this new raised Power being compared with the Given Power, will shew whether it have a true Root or not; which if it have, then it's easily evolved: If not, then it is a Surd Compound, and must have its Sign prefixed to it, and is no otherwise to be expressed; till it come to be evolved in Numbers.

But because such Operations are tedious, I shall omit giving Examples thereof in Species, and according to the Design of this Tract, (which is brevity,) proceed to Fractions.

CHAP. IV.

Of Fractions, or Broken Quantities.

§. 1. *What is done about vulgar Fractions in Numbers, the same may also be performed in Quantities.*

Rule 1. **T**O bring whole Quantities to a Fraction of a Given Denominator, multiply both together for a Numerator, and place the Given Quantity for the Denominator; thus, $a = \frac{ad}{d}$, and $b = \frac{bd}{d}$, also $a = \frac{ac+da}{c+d}$, $d + \frac{dd}{b} = \frac{db+dd}{b}$ &c.

Rule 2. When whole Quantities are to be expressed Fraction-wise, give them an Unity for a Denominator, as $aa = \frac{aa}{1}$ and $ad + bc = \frac{ad+bc}{1}$.

§. 2. *To reduce Fractions to a lower Denomination.*

Rule 1. **W**Hen the same Quantities are found both in the Numerator and Denominator, cast them away in both, and a Fraction remains of the same value, as $\frac{abc}{abc} = \frac{a}{d}$, and $\frac{bcdf}{bdf} = c$. $x + \frac{adf}{df} = x + a$.

Thus the greatest common Divisor brings the Fraction to the lowest Denomination.

Rule.

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Rule 2. But when the Fraction is a Compound, the Method to obtain that common Divisor, is by dividing the Denominator by the Numerator, and the Numerator by the Residue, and so on (as in Vulgar Arithmetick,) till there remain nothing; by the last Divisor, divide both the Numerator and Denominator of the Fraction, and it will be reduced to the Lowest Denomination.

Example. Let the Fraction proposed be $\frac{ab + cb}{aa + 2ac + cc}$

	1	$aa + 2ac + cc$, the Denominator,
	2	$ab + bc$, the Numerator,
$1 \div 2$	3	$\frac{a + c}{b}$
$3 \times b$	4	$a + c$
		<i>Note, Any Fraction is Multiplied by so much as is cast away out of the Denominator.</i>
		Hence it appears $a + c$ is the greatest Divisor,
$2 \div 4$	5	$\frac{b}{a + c}$ the new Fraction
$1 \div 4$	6	$\frac{ab + cb}{aa + 2ac + cc}$

But if this way be thought too tedious, then discover all the Aliquate parts in each, and the Denominator and Numerator being divided thereby, are reduced.

But if after all means used, there cannot be found one common Divisor to both, then are the Fractions Prime to each other.

§. 3. To bring different Fractions into one Denomination, retaining the same value.

Rule 1. **T**his is perform'd (as in *Vulgar Fractions*) by Multiplying the Denominators into each other for a new Denominator, and the Numerators cross-wise into each others Denominator, for new Numerators.

Let

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Let the Fractions be, $\frac{a}{b}$, $\frac{c}{d}$, $\frac{f}{h}$, &c.

Reduced will stand thus, $\frac{adh}{bdh}$, $\frac{bch}{bdh}$, $\frac{bdf}{bdh}$ and

retain the same value with the first.

By these means Fractions are made fit for *Addition* and *Subtraction*, which Two Rules cannot well be perform'd, until different Fractions are reduced to one Denomination.

§. 4 Addition of Fractions.

Rule. **W**hen the Fractions are brought to one Denomination, (as before) add, or subtract their Numerators.

Example,

Thus,

1	$\frac{3b}{d}$	$\frac{3c + 2b}{d + f}$	$\frac{a - b + de}{c + f}$
2	$\frac{a}{d}$	$\frac{x + b}{d + f}$	$\frac{a + b - de}{c + f}$
1 + 2	$\frac{a + 3b}{d}$	$\frac{3c + 2b + x + b}{d + f}$	$\frac{2a}{c + f}$

Subtraction.

Thus,

1	$\frac{3b}{d}$	$\frac{3c + 2b}{d + f}$	$\frac{a - b + de}{c + f}$
2	$\frac{a}{d}$	$\frac{x - b}{d + f}$	$\frac{a + b - de}{c + f}$
1 - 2	$\frac{3b - a}{d}$	$\frac{3c + 2b - x + b}{d + f}$	$\frac{2de - 2b}{c + f}$

§. 5. Mul-

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§. 5. Multiplication of Fractions.

Rule 1. **M**ultiply the Numerators and Denominators respectively into each other, (as in Vulgar Fractions) their Products are the new Fractions.

Note, But first prepare them for the work, by making mix'd Quantities improper Fractions, having first brought down the Fractions to their lowest terms; also make whole Quantities like Fractions, by placing an Unity under them: *As in the first Section of this Chapter.*

1	$\frac{2a}{b}$	$\frac{2a + b}{d - e}$	$\frac{a - 7 + b}{d}$
2	$\frac{3c}{d}$	$\frac{de}{d + e}$	$\frac{a + 8}{f - g}$
1 × 2	3	$\frac{6ac}{db}$	$\frac{2ade + dbe}{dd - ee}$
			$\frac{aa + a + ba + 8b - 56}{df - dg}$

Whole Quantities by Fractions stand as followeth.

Thus,	1	$\frac{b + c}{1}$	$\frac{ab - ad}{1}$	$\frac{ac + 7}{1}$
	2	$\frac{cd}{f}$	$\frac{b}{c}$	$\frac{a + c}{b + c}$
1 × 2	3	$\frac{bcd + ccd}{f}$	$\frac{abb - abd}{c}$	$\frac{aac + 7a + acc + 7c}{b + c}$

§. 6. Division of Fractions.

Rule 1. **D**ivision is thus perform'd: Multiply the Numerator of the Dividend, by the Denominator of the dividing Fraction, for a new Numerator, and the other Numerator and Denominator, for a new Denominator.

Suppose

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Suppose $\frac{a+b}{d}$ were to be divided by $\frac{c-b}{f}$,

it will stand thus $\frac{a+b}{d} \div \frac{c-b}{f} = \frac{af+bf}{cd-bd}$

Again, Admit $aa+bc$, were to be divided by $\frac{cb-ad}{c+g}$

it will be $\frac{aa+bc}{1} \div \frac{cb-ad}{c+g} = \frac{aac+bcc+aag+bcg}{cb-ad}$

Rule 2. When the Fractions are of one Denomination, divide the Numerators, and cast off the Denominator ;

as $\frac{aacc}{b} \div \frac{ac}{b} = ac$, for $\frac{aaccb}{acb} = ac$.

Again let $\frac{abb}{d} \div \frac{abc}{c} = \frac{b}{d}$ for $\frac{abbc}{abcd} = \frac{b}{d}$.

§. 7. Involution of Fractions, is performed in all respects like Multiplication.

Thus,	1	$\frac{3a}{b}$	$\frac{4bd}{c}$	$\frac{a+b}{g+x}$
1 2	2	$\frac{9aa}{bb}$	$\frac{16bbdd}{cc}$	$\frac{aa+2ab+bb}{gg+2gx+xx}$
1 3	3	$\frac{27aaa}{bbb}$	$\frac{64bbbddd}{ccc}$	$\frac{a^3+3baa+3bba+b^3}{g^3+3ggx+3gxx+x^3}$

N. B. If after any of these Operations in the foregoing parts of Addition, Subtraction, &c. there be like Quantities in the Numerator and Denominator, care is to be taken that such Fractions be reduced lower, by some of the former Rules, if possible, before they be engaged in other Operations.

I shall pass over Evolution, because it rarely falls out, that both the Numerator and Denominator have a true Root; therefore if the Sign of the Root be prefixed, it is sufficient, until such Fractions are evolved in Numbers.

CHAP.

CHAP. V. Of Proportions.

§ 1. **P**roportion is either Disjunct, or Continued; Disjunct is, when of Four Quantities, the First is to the Second, as the Third is to the Fourth; if Quantities be thus qualified, then the Product of the two Extrems, is equal to the product of the two Means. And if the product of the Extrems be equal to the product of the Means, then are those Quantities proportional, (16 Encl. 6.) let the Four Quantities proposed be a, b, c, d , if these be proportional, Oughtred's *Clavis*, Chap. 6. they shall also be proportional, being *Alterned, Inverted, Compounded, Divided, Converted, and in Mixture.*

1, ∴	1	$a : b :: c : d$, Direct.
	2	$ad = bc$
3, ∴	3	$a : c :: b : d$, Alterned.
	4	$ad = bc$
5, ∴	5	$b : a :: d : c$, Inverted.
	6	$bc = ad$
7, ∴ Or 9, ∴	7	$a + b : b :: c + d : d$, Compounded.
	8	$ad + bd = bc + bd$, that is, $ad = bc$.
	9	$a + c : c :: b + d : d$.
	10	$ad + cd = bc + cd$, that is $ad = bc$.
11, ∴ Or 13, ∴	11	$a - b : b :: c - d : d$, Divided.
	12	$ad - bd = bc - bd$, that is, $ad = bc$.
	13	$a - c : c :: b - d : d$.
	14	$ad - cd = bc - cd$, that is, $ad = bc$.
15, ∴	15	$a : b ± a :: c : d ± c$, Converted.
	16	$ad ± ac = bc ± ac$, that is, $ad = bc$.
17, ∴ 18, ±	17	$a + b : a - b :: c + d : c - d$, in Mixture.
	18	$ac - ad + bc - bd = ac + ad - bc - bd$
	19	$2bc = 2ad$, that is, $ad = bc$.

From

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From the foregoing Operations, it is evident, that Four proportional Quantities may be varied, and intermingled one with the other, and yet retain their true Proportions; which variation may be of good use upon several occasions.

N. B. And from hence may be learnt, how to convert Analogies into *Æquations*, or the contrary of Converting *Æquations* into Analogies.

§. 2. Continued Proportion (or Geometrical Progression) is, when of Three, Four, Five, or more Quantities, the First is to the Second, as that Second is to the Third; and the Second is to the Third, as that Third is to the Fourth, &c. that is, when all the Middle Terms between the First and Last Term are both Consequents and Antecedents; such are in $\frac{b}{a} :: \frac{c}{b} :: \frac{d}{c} :: \frac{e}{d}$.

as, $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa} :: \frac{bbb}{aa} : \frac{bbbb}{aaa}$, &c. these

are in continued Proportion.

Much may be said of the Nature of *Proportions*, for indeed the whole Body of the *Mathematicks*, is nothing but a Complete Compassion thereof. *composition*

§. 3. Before we leave this, take one Consideration more, that if never so many Quantities be Proportional, it will be, as one of the Antecedents is to its Consequent, so is the Sum of the Antecedents, to the Sum of the Consequents. As, $a : b :: c : d :: f : g :: h : k$, &c. Then will $a : b :: a + c + f + h : b + d + g + k$, &c.

D

CHAP.

CHAP. VI.

Of Equations, and Questions resolved thereby.

§. 1. **Q**uantities are said to be brought to an Equation, when some one of those that be sought, (or some Powers thereof, as Squares, Cubes, &c.) is found equal to those that be given. And this is to be obtained by a mutual comparing of equal Quantities, (or things) with equal, by such Methods or Rules, as the Nature of the Problem (or Question) requireth.

Therefore, when any Question is proposed to be resolved, it is highly requisite that the Design or Meaning thereof, be fully and clearly comprehended, that so the Question may be truly stated, or fitted for Operation.

And this part of the work is something difficult; yet having a competent knowledge in the Principles of *Geometry* and *Arithmetick* (together with a good and desiring ingenious,) a little Practice will render it facile and pleasant.

Having got a clear understanding of the Question proposed, for each Quantity concerned therein, be it a Line, or Number, put or substitute a Letter, as before taught at the beginning of this Tract, (*Page 2, and 3*;) but if Two, or more Quantities in the Question are granted to be equal, they may be expressed by the same Letter.

Having placed down all the concerned Quantities in their due order, (according to the import of the Question) then, consider whether the same be limited or not; for the discovery thereof, observe the following Rules.

Rule

Rule 1. Whensoever the Number of required Quantities exceeds the Number of the given *Æquations*, the Question is capable of innumerable Answers.

For Example, Suppose a Question were thus proposed. There be Three Numbers, the First is equal to the Square of the Second, and the Sum of the First and Second, is equal to the Third, What are they Three Numbers. To signify these Three Numbers, put Three Letters; call the First a , the Second e , and Third y ; then $a = ee$, and $a + e = y$, according to the Question. Here the sought Quantities are Three, to wit, a , e , and y , but the given *Æquations* are but Two; therefore this Question is unlimited. Because for any one of these Three Quantities, any Number may be taken at pleasure. Therefore, unlimited.

Rule 2. But when the given *Æquations*, (not depending one upon the other, are as many as the required Quantities, then hath the Question a determinable number of Answers; for then each Quantity concern'd, hath but one single value.

And if the given *Æquations* be more than the sought Quantities, they not only limit the Question, but oftentimes render it unresolvable, by being inconsistent one with another.

The Question be^{ing} stated, and found limited to one Answer, (or at least so bounded, as to have a certain determinable number of Answers,) let all the Quantities be so Ordered, Moulded or Compared, by Adding, Subtracting, Multiplying, or dividing them, until there remain but one unknown Quantity, (or some Powers thereof) equal with the given Quantities. And all the rest of the unknown Quantities concern'd in the Question, are cast off or vanished; which indeed is the chiefest business required in the forming of an *Æquation*, and therefore ought to be well considered, but no Rules can be prescribed to suit all Cases.

And next, because that in most (if not all) *Equations*, when first they appear out of the covers of the *Question*, the known and unknown *Quantities* are intangled and mixt together, on both sides of the *Equation*; it is therefore requisite so to order and clear such *Equations*, that one side thereof may consist of known *Quantities* only, and the other side of the unknown *Quantities*, (still keeping them to a just equality,) which work fits the *Question* for a *Solution* in *Numbers*, and is not improperly called *Reduction*.

For the performance thereof, observe the following *Rules*, wherein most, or all the parts of *Algebraick Arithmetick*, are concern'd, or employed.

By Addition.

Rule 1. **W**hen any *Quantity* is connected with the Sign —, it is added, by casting away the Sign —, and transferring the *Quantity* to the other side of the *Equation*, with the Sign +

$$\begin{array}{l|l} \text{Thus,} & 1 \quad a - b - 15 = 60 \\ 1 + 15 & 2 \quad a - b = 75 \\ 2 + b & 3 \quad a = 75 + b \end{array}$$

$$\begin{array}{l|l} \text{Again,} & 1 \quad aa - bc - d = cc - ba \\ 1 + ba & 2 \quad aa + ba - bc - d = cc \\ 2 + bc & 3 \quad aa + ba - d = cc + bc \\ 3 + d & 4 \quad aa + ba = cc + bc + d \end{array}$$

By Substraction.

Rule 2. When any *Quantity* is Connected with the Sign +, it is to be Subtracted, by casting away the Sign

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Sign $+$, and transferring the Quantity to the other side of the Equation, with the Sign $-$.

$$\begin{array}{l|l} \text{Thus,} & 1 \quad a + b + c = d \\ 1 - c & 2 \quad a + b = d - c \\ 2 - b & 3 \quad a = d - c - b \end{array}$$

These Two Rules are called Transposition of Quantities.

By Multiplication.

Rule 3. By this Rule, part, (or the whole) being in Fractions, are brought into whole Quantities.

$$\begin{array}{l|l} \text{Thus,} & 1 \quad a + b = \frac{dd}{a - b} \\ 1 \times a - b & 2 \quad aa - bb = dd \\ 2 + bb & 3 \quad aa = dd + bb \end{array}$$

By Division.

Rule 4. When there is any Quantity known or unknown concerned in every Member of an Equation, Divide the whole Equation by such Quantity, and it will clear one Member thereof (at least) and bring down the rest.

$$\begin{array}{l|l} \text{Thus,} & 1 \quad ab + bca = bd \\ 1 \div b & 2 \quad aa + ca = d \end{array}$$

$$\begin{array}{l|l} \text{Again,} & 1 \quad ab + ca = da \\ 1 \div a & 2 \quad ab + c = d \\ 2 - c & 3 \quad ab = d - c \\ 3 \div b & 4 \quad a = \frac{d - c}{b} \end{array}$$

D 3

Again,

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Again,	1	$baaa + bbaa - ca = da$
1 + a	2	$baa + bba - c = d$
2 + c	3	$baa + bba = d + c$
3 ÷ b	4	$aa + ba = \frac{d + c}{b}$

By Involution.

Rule 5. When there is an Equation between like Surds, (or Radical Signs) cast away the Radical Signs, and the Quantities are thereby reduced to Rational ones.

Thus,	1	$aa\sqrt{b} = b\sqrt{da}$
1 ⊗ 2	2	$aaaab = bbda$
2 ÷ ab	3	$aaa = bd$

Again,	1	$b\sqrt[3]{aa} = \frac{dc}{b}$
1 ⊗ 3	2	$bbbaa = \frac{dddccc}{bbb}$
2 ÷ b ³	3	$aa = \frac{dddccc}{bbbbb}$

Also,	1	$\sqrt[3]{baa + 3ba - d} = \frac{b + d}{2}$
1 ⊗ 2	2	$\frac{baa + 3ba - d}{4} = \frac{bb + 2bd + dd}{4}$
2 × 4	3	$baa + 3ba - d = bb + 2bd + dd$
3 + d	4	$baa + 3ba = bb + 2bd + dd + d$
4 ÷ b	5	$aa + 3a = \frac{bb + 2bd + dd + d}{b}$

By Evolution.

Rule 6. By this Rule *Æquations* are oft brought down to lower Powers, thus,

$$\begin{array}{l|l} 1 & aa + 2ba + bb = d \\ 2 & a + b = \sqrt{d} \end{array}$$

But if the *Æquation* had been $aa + 2ba = d$, which is an imperfect Square, and wants the Square of bb to fill it up, before it can be brought down as above; such Operation Dr. Pell calls, *Compleating the Square*; and when it is performed, useth this Sign \square put in the Margin.

$$\begin{array}{l|l} \text{As,} & 1 \quad aa + 2ba = d \\ 1, \square & 2 \quad aa + 2ba + bb = d + bb \\ 2 \text{ w } 2 & 3 \quad a + b = \sqrt{d + bb} \\ 3 - b & 4 \quad a = \sqrt{d + bb} - b. \end{array}$$

$$\begin{array}{l|l} \text{Suppose,} & 1 \quad aa - ba = d \\ 1, \square & 2 \quad aa - ba + \frac{1}{4}bb = d + \frac{1}{4}bb \\ 2 \text{ w } 2 & 3 \quad a - \frac{1}{2}b = \sqrt{d + \frac{1}{4}bb} \quad + \frac{1}{2}b = \sqrt{d + \frac{1}{4}bb} \\ 3 + \frac{1}{2}b & 4 \quad a = \sqrt{d + \frac{1}{4}bb} + \frac{1}{2}b. \end{array}$$

In these it is evident, that the last place of bb , or that of $\frac{1}{4}bb$, is produced, by involving half the Coefficient b , of the Middle place.

§. When the *Æquations* be thus cleared or reduced, then are they fit to be resolved into Numbers, and are either Pure and Simple, or Mixt and Adfectèd. Pure and Simple *Æquations*, are such as have the unknown Quantity, (a) at one side thereof,

As $a = b + c$, or $a + b = d$, &c.

Alfo

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also when there is only some Power of (*a*),

As $aa = b$, and $aaa = c$, or $aaaa = d$, &c.

Mixt And Adfectèd, are such as have the unknown Quantities at one side of the *Æ*quation, in different Degrees, or Powers thereof,

As $aa + ba = d$, and $aa - ba = d$, or $ba - aa = d$.

these are called the Three Forms of Square Adfectèd *Æ*quations.

Again, $aaa + ba = c$, and $aaa - ba = c$, or $ba - aaa = c$,

these are called the Three Forms of Cubick Adfectèd *Æ*quations.

Infinite other Varieties there are, such

As $aaa + ba + ca = d$, and $aaa - ba = d$.

Or $aaaa + baaa + caa - da = f$, &c.

How these, or any other, of all Degrees, or howsoever Mixt and Adfectèd, may be easily resolvèd into Numbers, shall be effectually shewèd further on, when we come to the Resolving of such Problems as will produce the same; taking each in their Order. *viz.* First, I will shew how to Resolve, (or Extract the Roots of) all pure *Æ*quations; next the Three Forms of Square Adfectèd *Æ*quations, the like in Adfectèd Cubicks, and so on to others of different degrees.

But before we proceed to that part, it will be convenient to insert a few easie Questions in Numbers, thereby to explain, and (in part) shew the use of the foregoing Rules. The which will be as an Introduction to the succeeding Problems.

CHAP.

CHAP. VII.

Containing several easie Questions in Numbers fit for a Learners Practice.

Quest. I. **T**Here be Two unknown, and unequal Numbers, whose Sum is $53 = s$, and their Difference is $21 = d$;

What are those Numbers?

For the Greater put a , and for the Lesser put e .

Then according to the Question, they will stand in their Substitutes thus,

		Quantities.	Numbers.
	1	$a + e = s$	$= 53$
	2	$a - e = d$	$= 21$
$1 + 2$	3	$2a = s + d$	$= 74$
$3 \div 2$	4	$a = \frac{s + d}{2}$	$= \frac{74}{2} = 37$
$1 - 2$	5	$2e = s - d$	$= 32$
$5 \div 2$	6	$e = \frac{s - d}{2}$	$= 16$

Hence it appears, that 37, and 16, are the Two Numbers sought.

Note, When any Number is placed in the Margin, with a Line over it, as $\bar{2}$ in the Fourth and, Sixth Steps, such is an absolute Number, and hath no relation to those Numbers in the small Column.

Here you may perceive, that the first Work is to cast off one of the Unknown Quantities; and because the Quantity e , hath both Signs, $+$ and $-$ prefix to it, therefore by the Rule of Addition, it may be vanished, as appears by the Work of the Third Step.

Quest.

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Quest. 2. There be Two Numbers (as before) whose Sum is $56 = s$, and the Greater (a) divided by the Lesser (e) the Quotient is $2,5 = q$, What are the Numbers? a , and e .

	1	$a + e = s$	} by the Questions.
	2	$a \div e$, or $\frac{a}{e} = q$	
$3 - a$	3	$e = s - a$	
$2 \times e$	4	$qe = a$	
$4 \div q$	5	$e = \frac{a}{q}$	
$3, 5,$	6	$s - a = \frac{a}{q}$	
$2 \times q$	7	$qs - qa = a$	
$7 + qa$	8	$qa + a = qs$	
$8 \div q + 1$	9	$a = \frac{qs}{q+1}$ for $q+1 \times a = qa + 1a$	
$3 - 9$	10	$e = s - \frac{qs}{q+1} = \frac{s}{q+1}$	

For an Explanation, take the same in Numbers.

	1	$a + e = 56$	
	2	$a \div e$, or $\frac{a}{e} = 2,5$	
$3 - a$	3	$e = 56 - a$	
$2 \times e$	4	$2,5e = a$	
$4 \div 2,5$	5	$e = \frac{a}{2,5} = 56 - a$	
$5 + a$	6	$a + \frac{a}{2,5} = 56$	
$5 \times 2,5$	7	$2,5a + a = 140$	
$7 \div 3,5$	8	$a = \frac{140}{3,5} = 40$ That is, by $2,5 + 1$	
$3,$	9	$e = 56 - 40 = 16$	

Arithmetical Questions. 35

Quest. 3. Let there be given the Difference of the afore-
said Two Numbers, to wit, $24 = d$, and the Quotient,
 $2,5 = q$, from thence to find out the Two Numbers.

	1	$a - e = d$	} by the Question,
	2	$\frac{a}{e} = q$	
1 + e	3	$a = d + e$	
2 x e	4	$a = qe$	
3, 4,	5	$qe = d + e$	
5 - e	6	$qe - e = d$	
8 ÷	7	$e = \frac{d}{q-1}$	
1 + 7	8	$a = d + \frac{d}{q-1} = \frac{qd}{q-1}$	

The reducing of this into Numbers, is easie, (the fore-
going Questions considered.)

Quest. 4. There be Two Numbers, their Sum is $56 = s$,
and the Difference of their Squares is $1344 = x$; What
are they Numbers?

	1	$a + e = s$	} by the Question,
	2	$aa - ee = x$	
2 ÷ 1	3	$a - e = \frac{x}{s}$ for $a + e \times a - e = aa - ee$	
1 + 3	4	$2a = s + \frac{x}{s} = \frac{ss + x}{s}$	
4 ÷ 2	5	$a = \frac{ss + x}{2s}$	
1 - 3	6	$2e = s - \frac{x}{s} = \frac{ss - x}{s}$	
6 ÷ 2	7	$e = \frac{ss - x}{2s}$	

Hence (a) is found equal to 40, and (e) equal to 16 by
the 5th and 7th Steps.

Quest.

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Quest, 5. Two Men have each a Sum of Money, the one hath Four times as much as the other; both their Sums put together, will not make 100 *l.* but if they be doubled, and from that double be taken 30 *l.* the remainder will be twice as much above 100 *l.* as before they wanted of 100 *l.* How much hath each Man?

Put *a* for the Greater, and *e* for the Lesser.

Then	{	1	$a = 4e$	} <i>d</i> , signifieth the difference their Sums wanted of 100 <i>l.</i>
		2	$a + e = 100 - d$	
And		3	$2a + 2e - 30 = 100 + 2d$, by the Question.	
2×2		4	$2a + 2e = 200 - 2d$	
$3 + 4$		5	$4a + 4e - 30 = 300$	
$5 + 30$		6	$4a + 4e = 330$	
1, 6,		7	$5a = 330$	
$7 \div 5$		8	$a = \frac{330}{5} = 66$	
1, 8,		9	$4e = 66$	
$9 \div 4$		10	$e = \frac{66}{4} = 16,5$	

Hence it is found, that one Man had 66 *l.* and the other 16 *l.* 10 *s.* which Two Sums will answer the Question in each particular.

Note, That as *e*, in the first Question, was cast off by *Addition*, (for the Reasons there mentioned,) so in the Second, the like was done by *Multiplication* and *Division*; in the Third, by *Multiplication* and *Addition*; in the Fourth, by *Division* and *Addition*; and in this Question by *Multiplication* and *Equality*.

Now if the Reason of these be considered, and once understood, the whole business of forming any *Equation*, will be easie; for the only *Mystery* therein, lies in casting off all the unknown Qualities but one, (of which, see *Pag. 27.*)

Quest

Arithmetical Questions. 37

Quest. 6. Three Men part their Money in this Manner; the First gives to the Second and Third, as much as they have about them, the Second gives to the First and Third as much as now they have, and the Third doth the like to the First and Second; after they had done thus, each Man had 20 Shillings. The Question is, what each Man had at first?

For the First Man's Sum put a , for the Second e , and for the Third y .

Then	{	1	$a - e - y$	{ is what the First hath left after his Gift.	
		2	$2e$	{ is what the Second and Third have, after the First Man's Gift.	
		3	$2y$		
$2 - 1 - 3$		4	$3e - a - y$	{ the First and Third doubled by the Second.	
1×2		5	$2a - 2e - 2y$		
3×2		6	$4y$		
$6 - 4 - 5$		7	$7y - a - e$	{ and so much each hath at last, viz. = 20.	
5×2		8	$4a - 4e - 4y$		
4×2		9	$6e - 2a - 2y$		
That is	{	10	$7y - a - e = 20$		
		11	$4a - 4e - 4y = 20$		
		12	$6e - 2a - 2y = 20$		
		13	$a - e - y = 5$		
		14	$3e - a - y = 10$		
		15	$2e - 2y = 15$		
		16	$6y - 2e = 25$		
		17	$4y = 40$		
		18	$y = 10$		
		19	$2e = 15 + 2y = 35$		
		20	$e = \frac{35}{2} = 17,5$		
		21	$a = 32,5$		

Ans. The First Man had 32 Shillings, 6 Pence, the Second had 17 Shillings, 6 Pence, and the Third had 10 Shillings.

E

Quest.

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Quest. 7. There be Three Numbers with these Properties, viz. if to the First and Second, be added half the Third, that Sum will be $= s$, (suppose 46, or any Number at pleasure,) and if one Third part of the First be added to the Second and Third, that Sum $= s$, as before; also if a Fourth part of the Second, be added to the First and Third, that Sum will be $= s$, as was the others. What are these Numbers?

For they three Numbers, put a , e , and y .

Then $\left\{ \begin{array}{l} 1 \quad a + e + \frac{1}{2}y = s \\ 2 \quad \frac{1}{3}a + e + y = s \\ 3 \quad a + \frac{1}{4}e + y = s \end{array} \right\} \left\{ \begin{array}{l} \text{stated according to the} \\ \text{Question.} \end{array} \right.$

$\left. \begin{array}{l} 1 \times 2 \\ 2 \times 3 \\ 3 \times 4 \end{array} \right\} \left\{ \begin{array}{l} 4 \quad 2a + 2e + y = 2s \\ 5 \quad a + 3e + 3y = 3s \\ 6 \quad 4a + e + 4y = 4s \end{array} \right\} \left\{ \begin{array}{l} \text{Thus all are} \\ \text{brought out of} \\ \text{their Fractions} \end{array} \right.$

$4 + 6 \quad 7 \quad 6a + 3e + 5y = 6s$

$7 - 5 \quad 8 \quad 5a + 2y = 3s$

$8 - 5s \quad 9 \quad 2y = 3s - 5a$

$9 \div 2 \quad 10 \quad y = \frac{3s - 5a}{2}$

$4 + 5 \quad 11 \quad 3a + 5e + 4y = 5s$

$11 - 6 \quad 12 \quad -a + 4e = s$

$12 + a \quad 13 \quad 4e = s + a$

$13 \div 4 \quad 14 \quad e = \frac{s + a}{4}$

$10 + 14 \quad 15 \quad e + y = \frac{3s - 5a}{2} + \frac{s + a}{4} = \frac{7s - 9a}{4}$

$15 \times 3 \quad 16 \quad 3e + 3y = \frac{21s - 27a}{4}$

$5, 16, \quad 17 \quad a + \frac{21s - 27a}{4} = 3s$

$17 \times 4 \quad 18 \quad 4a + 21s - 27a = 12s$

$18 + \quad 19 \quad 23a = 9s, \text{ if } (s) \text{ be put } = 46, \text{ then}$

$19 \div 23 \quad 20 \quad a = \frac{9s}{23} = 18, \text{ and } e = 16$

Arithmetical Questions. 39

Quest. 8. There be Four Numbers, their Sum is $= s$, (any Number at pleasure, suppose 100) and if to the First be added $b = 7$, (or any Number,) from the Second be Subtracted b , the Third Multiplied by b , it's required, that such Sum, Difference, Product and Quotient be equal to each other. What are the Four Numbers?

For the First put a , for the Second e , for the Third y , and for the Fourth u .

Then	1	$a + e + y + u = s$	} according to the Propositions in the Question.
and	2	$a + b = e - b$	
	3	$y b = \frac{u}{b} = a + b$	
$1 + b$	4	$a + 2b = e$	
$3 \div b$	5	$y = \frac{a + b}{b}$	
$3,$	6	$\frac{u}{b} = a + b$	
$6 \times b$	7	$u = ba + bb$	
$1, 4, 5, 7,$	8	$a + a + 2b + \frac{a + b}{b} + ba + bb = s$	
8,	9	$bb a + 2ba + a = bs - b^3 - 2bb - b$	
$9 \div$	10	$a = bs - bbb - 2bb - b \div bb + 2b + 1$	
		Suppose $s = 100$, and $b = 7$	
then		$bs - bbb - 2bb - b = 252$	
		$bb - 2b + 1 = 64$	
4,	11	$a + 2b = 17,9375 = e$	
5,	12	$\frac{a + b}{b} = \frac{10,9375}{7} = 1,5625 = y$	
7,	13	$ba + bb = 76,5625 = u$	

Ans. The First Number is 3,9375, the Second is 17,9375, the Third is 1,5625, and the Fourth is 76,5615.

40 Arithmetical Questions.

I have made choice of these few plain and familiar Questions, purely for the Young Learner's Exercise, and have placed them in such Order, as may lead him gradually into this most excellent Art of *Æquations*. I have likewise trased them from Step to Step, thereby to explain and render each Process easie, (even to the meanest Capacity,) which I doubt not but with a little Consideration, will easily be understood; and then he may proceed to the Resolving others, of a different and more difficult Nature, of which there are great varieties in several Authors, to whom I refer those that are desirous of such Speculations; and proceed, (according to my Design) of Extracting the Roots (in Numbers) of all pure *Æquations*; then to the Resolving of such Problems only, as will raise different Degrees of Mixt or Adfectèd *Æquations*; and there shew how the same may be resolved into Numbers.

Which last Work hath heretofore been a Business of great Difficulty and Labour; as the Studious in this Art no doubt have experienced. But the same is now rendered much easier, and made practicable, by a new Method of Converging Series, invented by the Learned and Ingenious Mr. *Joseph Raphson*, Fellow of the *Royal Society*; and Published in a *Latine* Treatise, by the Title of *Analysis Æquationum Universalis*, Printed 1690; wherein he hath given (and demonstrated) general Theorems for the Solving (or Extracting the Roots) of all manner of *Æquations*, howsoever Mixt or Adfectèd.

I shall here give a small Abstract thereof, and refer the more inquisitive Reader to the Book it self; where (I doubt not, but) he will meet with ample satisfaction.

C H A P.

CHAP. VIII.

An Abstract of Mr. Raphson's Method of Converging Series.

IN this Abstract, I shall pass over the Author's Demonstration, giving only Two Examples of the Method, explaining the same; then insert a few of his Theorems, with some Remarks upon the whole, and so conclude it.

Example in the Square Root.

Viz. $aa = c$ { Any Given Number whose Square Root is required.

What is the Value of the Root a ?

Let the Root a , be supposed to be divided into Two Parts, to express each of which, put or substitute Two Letters, viz. $g + x = a$;

Then is, $gg + 2gx + xx = aa = c$.

He rejecteth the power of x , (viz. xx ,) and then it becomes $gg + 2gx = c$; which is $2gx = c - gg$, and from hence ariseth this

$$\text{Theorem, } x = \frac{c - gg}{2g}$$

Note, The Number signified by g , may be taken at pleasure; but if g , be made the first Root, or Single side of the Given Square, the sooner will the other Member, (or Converging part x ,) Converge to the true Root.

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To explain what is here said, take this Example,

$as = c = 2$. As in Analysis Aequationum, Pag 5.

take the First $g = 1$.

$$x = \frac{c-g}{2}$$

$$\text{Then } \left\{ \begin{array}{l} c = 2 \\ -gg = 1 \end{array} \right\} = 1$$

$$\text{And } 2g = 2) \quad \underline{1,0} \quad (,5 = x$$

To the First $g = 1$, add $,5 = x$

$$+ \frac{,5}{1,5}$$

the new $g = 1,5$ for a Second Operation.

$$\text{Then } \left\{ \begin{array}{l} c = 2 \\ -gg = 2,25 \end{array} \right\} = -0,25$$

$$\text{And } 2g = 3) \quad \underline{-0,25} \quad (-,083 = x$$

Last $g = 1,5$ from which must now be taken the Converging Number x , because it hath the Sign — prefix to it.

$$\begin{array}{r} 1,5 \\ -,083 \end{array}$$

the new $g = 1,417$ for a third Operation.

Then $c = 2$

$$-gg = 2,007889$$

$$\text{and } 2g = 2,834) \quad \underline{-,007889} \quad (-,002783 = x$$

$$\begin{array}{r} 1,417 \\ -,002783 \end{array}$$

$$g = 1,414217$$

Or if more accuracy be required, it may be called a New g , for a Fourth Operation; and by repeating the Operations, you may have as many Places in the Root as you please.

Next

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Then $\begin{cases} d = 37945 \\ -ggg = 37966,934881 \end{cases}$
 and $3gg = 3388,8963) \quad -21,934881(- ,006472 = x$

Last $g = 33,61$ from which must now be taken the $- ,006472 = x$
 hence, $a = 33,603528$

Or if more exactness be required, then it may be called a new g , as before, &c.

I shall desist giving more Examples in Numbers, this being sufficient to shew the Method of proceeding in each Equation; and only insert a Table or Two of the several Theorems, as they are recorded in Mr. Raphson's Book before mentioned.

The Given Powers, whose Root is required.	First Theorems.	Second Theorems.
For the Square Root $aa = c$	$x = \frac{c - gg}{2g}$	$a = \frac{c + gg}{2g}$
For the Cube $aaa = d$	$x = \frac{d - ggg}{3gg}$	$a = \frac{d + 2ggg}{3gg}$
For the Biquadr. $aaaa = f$	$x = \frac{f - gggg}{4ggg}$	$a = \frac{f + 3gggg}{4ggg}$
For the Surfolide $a^5 = f$	$x = \frac{f - ggggg}{5gggg}$	$a = \frac{f + 4ggg}{5gggg}$

By what is here set down, it is obvious how to proceed for Extracting the Root (by this Method) of any simple Power, not only of these here inserted, but of any how high soever it be.

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A Table of Theorems, for the Resolving of Affected Equations, wherein I retain only those with the Converging (x,) and have omitted the Second sort, which Converge by the (a) it self; such as in the Second sort of the foregoing Table.

Equations proposed.	Theorems.
$aa + ba = c$	$x = \frac{c - gg - bg}{2g + b}$
$aa - ba = c$	$x = \frac{c + bg - gg}{2g - b}$
$ba - aa = c$	$x = \frac{c + gg - bg}{b - 2g}$
$aaa + ca = d$	$x = \frac{d - ggg - cg}{3gg + c}$
$aaa - ca = d$	$x = \frac{d + cg - ggg}{3gg - c}$
$ca - aaa = d$	$x = \frac{d + ggg - cg}{c - 3gg}$
$aaa + baa + ca = d$	$x = \frac{d - ggg - bgg - cg}{3gg + 2bg + c}$
$aaa + baa - ca = d$	$x = \frac{d - ggg - bgg + cg}{3gg + 2bg - c}$
$aaa - baa - ca = d$	$x = \frac{d + bgg + cg - ggg}{3gg - 2bg - c}$
$baa + ca - aaa = d$	$x = \frac{d + ggg - bgg - cg}{2bg + c - 3gg}$

This Table is continued in Mr. Raphson's Book to Four or Five Pages more, but for brevity sake I am forced to omit transcribing them.

There

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There be several things observable in this Method, amongst the rest, these are not the least, *viz.* That at each Operation, the Converging Number x , will double the last preceeding g , (or Number of Figures in the last Root, especially after the Second Operation, the Imperfection being only in the last Figure of the Root, so increased, which often proves too large, and therefore consequently the next Converging Number x , will have the Negative Sign —.

Also if there happen to be a mistake committed in any Operation, such mistake doth not destroy the preceeding work, for the same will be rectified (though it be not discovered) in the next succeeding Operation, unless it be very gross.

Again, it produceth the Roots of all Powers, ~~be~~ they never so high, in the same manner, and with the same exactness, as it doth those of a lower Rank, the respective Involutions being considered, which require to be always of the same height with the Given Powers; and the Divisors of the next Inferiour, or lower Powers.

These high Involutions and large Divisors, have often caused me to wish, that some means might be used of improving this Method, (or some lucky Discovery made of an other) such as would produce the several Resolvends and Divisors in lower Terms: And being (upon an extraordinary occasion) employed in the Calculation of several Problems, consisting of very high Affectèd Equations, made me the more desirous thereof, and put me upon thoughts, how (if possible) to accomplish the same; which if effected, would undoubtedly render the several Operations more easie.

Whilst I was ruminating upon this, there came into my Mind the following Method, the which hath not only answered my Desire in that particular, of producing Low Resolvends and Divisors, but even out-done my Expectation, in causing the Root to Converge quicker; for at each Operation, it will in most cases triple the Places of Figures in the last preceeding Root; in several it will do more, and in some cases, the Root Convergeth by a continued Series, without repeating the Operations at all, as will plainly appear in the Practice thereof.

CHAP.

CHAP. IX.

*Containing a new and general Method of
Extracting the Roots (in Numbers) of
all pure Equations.*

I Shall communicate this Method with all the plainness
(and brevity) I can, and therein shew the Process
used in raising and forming the Theorems for the
Solution of each Equation, as they come in their Order.

Now because there is required an Involution of the In-
creased Root, at the repeating of several Operations, (tho
not so high as the Given Power,) I will here insert an
easie Method of Composing the Square of an increased
Root, by having the Square and its Root, of one part
thereof, with the increasing Numbers of the other part of
the Root, performed without involving the whole Root
so increased.

Root =	7	4	9	3	8	0	5	2
Square of 7 =	49	16						
	28							
	28							
Square of 74 =	547	68	109					
		2247						
		2247						
		666						
		666						
Square of 7493 =	56145049	6400	2504					
		14987610						
		14987610						
		3746900						
		3746900						
		59944						
		59944						
Square of 74938052 =	5615711637552704							

This

This is so obvious, I suppose there needs no Explanation thereof: *The difference betwixt it and the common Method, I leave to the consideration of the Ingenious*

From the foregoing Genelis, or Composition of the Square, it is evident, that for each single Figure, (or Cypher in the Root, there will arise Two places of Figures in the Square; and from the like Composition of the Cube, it would be as evident there will arise Three, and in the Biquadrate Four, &c.

That is, there will arise so many Places of Figures in the Involved Number, for one in the Root, as the Index of such Power denotes.

Therefore when any Number is proposed to have its Root Extracted, the preparative work is to consider the Index of the Required Root, and to allow so many places of Figures in the Given Number, for each single Figure in the Root, as such Index shall denote; which Account must always take its beginning from the place of Unity, and ascend towards the Left-hand, if the Given Number be Integers, or descend towards the Right in Decimal parts.

Of the Index of any Root, see *Page 5*, and such Index is the same with that of the Power, viz.

$$\left. \begin{array}{l} aa = b \\ aaa = c \\ aaaa = d \\ aaaaa = f \end{array} \right\} \text{ is } \left\{ \begin{array}{l} a^2 = b \\ a^3 = c \\ a^4 = d \\ a^5 = f \end{array} \right\} \text{ then } \left\{ \begin{array}{l} a = \sqrt[2]{b} \\ a = \sqrt[3]{c} \\ a = \sqrt[4]{d} \\ a = \sqrt[5]{f} \end{array} \right.$$

And from hence is deduced the Method of distinguishing (or rather parting) the Given Numbers with Points, set over, or under their proper Figures.

For Example, Suppose any Number given,

As, 56430145789,723

out of which, if it be required to Extract any of these Roots,

Extraction of Roots.

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Roots, it must be pointed (according to the foregoing Considerations) in this manner, that is, for the

Square Root, 5 6 4 3 0 1 4 5 7 8 9, 7 2 3 0

Cube Root, 5 6 4 3 0 1 4 5 7 8 9, 7 2 3

Biquadrate Root, 5 6 4 3 0 1 4 5 7 8 9, 7 2 3 0

Surfsolide Root, 5 6 4 3 0 1 4 5 7 8 9, 7 2 3 0 0

And by these Points is known how many places of Figures there will be in the Root sought, for so many as are the Points, so many must be the Places of Figures in the Root. Now whether they be Integers or Decimals, is easily determined, by the Places of the Points over each. These premised, we may proceed to Examples in each.

Note, that in all the Theorems for resolving of Equations (or extracting of Roots) by the following Method, I make use of these Letters (*G*,) (*r*,) (*e*,) (*D*,) to represent each distinct part thereof, *viz.*

G, Always denotes the Given Resolvend, or known side of the Equation.

r, Represents the Root first taken, and when increased, &c.

e, Represents the Converging part of the Root; by which, *r*, is either increased or diminished, according as its Sign denotes, to wit, $+e$, or $-e$.

D, Is put for the Dividend, which is produced from *G*, being divided; and lessened by *r*, (into the Coefficients of Affected Equations) according as the Root requires; all which will plainly appear, and be easily understood, as we proceed on in the Examples.

I. For the Extracting of the Square Root, *viz.* $aa = G$.
What is the value of *a*?

In this Method (as in all others whatsoever) we suppose the Root *a*, to be Divided into Two parts, to
F express

express which, put the Two Letters before mentioned.

That is,	1	$r + e = a$
1 \odot 2	2	$rr + 2re + ee = aa = G.$
2 \div 2	3	$\frac{1}{2}rr + re + \frac{1}{2}ee = \frac{1}{2}G$
3 $- \frac{1}{2}rr$	4	$re + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr = D.$

And from hence will arise this

$$\text{Theorem } \sum \frac{D}{r + \frac{1}{2}e} = e.$$

First take an Example of a true Square Number.

$$\text{Let } aa = 4196837089 = G$$

$r = 6$, That is, the First Single Root, (or side) to 41

	$2098418544,5 = \frac{1}{2}G$
$-\frac{1}{2}rr = -18$	
$r = 6$) $298418544,5 = D (4 = e$
$+ e = 4$	$248 = re + \frac{1}{2}ee$
$r + e = 64783$	$50418 \quad (7 = e$
	$45045 = re + \frac{1}{2}ee$
but $r + e = a$	$53735 \quad (8 = e$
therefore	$51792 = re + \frac{1}{2}ee$
$a = 64783$	$194344,5 \quad (3 = e$
	$194344,5 = re + \frac{1}{2}ee$
	$(0,0)$

I presume the whole process of this is plain, however for the benefit of such as do not understand an Algebraick Theorem, let them take it in words thus,

First

Extraction of Roots.

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First point the Resolvend (or Given Number) as before taught; then take the greatest Square to the first Point thereof, out of the Resolvend to that Point; after which, half the remainder of the Resolvend, and point it anew; then make the Root of that Square so taken, a Divisor; inquiring how oft it may be found in the new Dividend, to the next Figure forward, reserving that Figure under the next Point; for the half Square of the Quotient Figure; which being found, and so placed, Multiply the Divisor by that Quotient; adding in the Tens, (if any arose from the half Square of the Quotient) as in plain Division, &c. Then annex the Quotient to the last Divisor, thereby making a new Divisor, with which proceed in all respects as with the last; and so on, until all be finished.

Value

If any seeming Difficulty appear, 'tis only in the true placing of the half Square of the Quotient Figure when it proves an odd Number; in that case bring down one Figure more of the next Period; under which, place the odd 5, which will always arise from the halving of the Square of an odd Number.

As for instance, Suppose 7, the Square thereof is 49, the half of which is 24, 5 to be placed as in the Second Operation of the last Example.

Note, That if the Given Number be a *Surd*, the Root r , will become a continued Series, *ad infinitum*; if it be still increased with the Converging Number e .

But if the Number of Places (in the Root of a *Surd* Number) be limited, the Root r , needs not be increased with e , to the last Operation of the whole Process; for the Work may be abbreviated, as in this Example. Suppose the Square Root of 3, (a *Surd* Number) were required to sixteen places of Figures therein.

F 2

Equation

Equation $aa = G$, and $G = 3$.

$$\begin{array}{rcl} r & = & 1, \\ + e & = & ,7 \end{array} \quad \begin{array}{rcl} 1,5 & = & \frac{1}{2} G \\ - 0,5 & = & \frac{1}{2} rr \end{array}$$

$$\begin{array}{r} r + e = 1,7320508 \quad 1,0000000000000000 = D \\ \underline{945} = re + \frac{1}{2} ee \quad (,7 = e \\ 550 \quad (3 = e \\ \underline{5145} = re + \frac{1}{2} ee \quad (2 = e \\ 3550 \quad (05 = e \\ \underline{3462} = re + \frac{1}{2} ee \quad (08 = e \\ 880000 \quad (138564032 = re + \frac{1}{2} ee \\ \underline{139875000} \quad 1310968 \end{array}$$

Having thus increased the Root to 8, or 9, Places by the Converging e , the rest may be obtained by plain *Division*, making the Root so found, a Divisor; but I also contract plain Division, by omitting, (or pricking off) one Figure of the Divisor at each Operation; and then the Work will stand thus,

$$\begin{array}{r} r = 1,7320508 \quad 131096800 \quad (07568877 = e \\ \underline{121243556} \\ 9853244 \\ \underline{8660250} \\ 1192994 \\ \underline{1039230} \\ 153764 \\ \underline{138560} \\ 15204 \\ \underline{13856} \\ 1348 \\ \underline{1211} \\ 137 \\ \underline{112} \end{array}$$

This Method of Contracting the Divisor, is of singular use upon several Occasions, where it may be admitted.

And what is here done as to this Abbridgment, may in like manner be performed in the Cube,

Last $r = 1,7320508$

$$+ e \quad 07568877$$

$$= 1,732050807568877 \text{ as was required.}$$

II. Of

Extraction of Roots.

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II. Of Extracting the Cube Root.

Equation $aaa = G$. To find the value of a ?

Put	1	$r + e = a$
1 \odot 3	2	$rrr + 3rre + 3ree + eee = aaa = G$
2 \div 3	3	$\frac{1}{3}rrr + rre + ree + \frac{1}{3}eee = \frac{1}{3}G$
3 \div r	4	$\frac{1}{3}rr + re + ee + \frac{1}{3}\frac{eee}{r} = \frac{1}{3}G \div r$

Let $\frac{1}{3}eee \div r$ be rejected, or cast off, as being of small Value, then it will become

$$\frac{1}{3}rr + re + ee = \frac{1}{3}G \div r.$$

and thence will $re + ee = \frac{1}{3}G - \frac{1}{3}rr = D$.

Hence ariseth this

$$\text{Theorem } \left\{ \frac{D}{r+e} = e. \right.$$

Let $37945 = G$, (as in *Pag 43* of this Tract.)

then $r = 30$, and $12648,333 = \frac{1}{3}G$

$r = 30$	$421,6111 = \frac{1}{3}G \div r$
$+e = 3$	$-300, = \frac{1}{3}rr$
$r+e = 33,6$	$121,6111 = D \quad (3 = e)$
	$99 = re + ee$
	2261
	$2016 = re + ee \quad (6 = e)$

New $r = 33,6$ $376,43849206 = \frac{1}{3}G \div r$

$-376,32 = \frac{1}{3}rr$	
$r+e = 33,6035262$	$11849206 = D \quad (003 = e)$
	$100809 = re + ee$
	$1768306 \quad (3 = e)$
	$1680175 = re + ee$

$n = 33,6035262$, &c.

Here you have the Cube Root Extracted to Nine Places, at Two easie Operations, the Defect being but Unity, in the last Place thereof.

$8813,100 \quad (2 = e)$
$6720804 = re + ee \quad (7 = e)$
$209249609 \quad (6 = e)$
201621156
$762844400 \quad (2 = e)$
672070524

F 3

This

S

54. Extraction of Roots.

This Method doth also Converge with the Negative Sign — if the value of e , happen to be taken too large; but to prevent that, take but one value of e , at the first Operation, and but Four at the Second; by adding, the Root will have but Six Places at Two Operations: But then a Third will produce Seventeen Places true; each Operation converging gradually, with the Affirmative Sign (which I take to be best.)

Take an Example of a true Cube Number, which I find in my Friend Mr. William Hunt's *Clarus Stereometria*, Page the 9th, viz.

$$aaa = 48627125 = G$$

$$\begin{array}{rcl} r = 300 & 16209041,66 & \text{O.c.} = \frac{1}{3}G \\ & 54030,1388 & \text{O.c.} = \frac{1}{3}G \div r \\ & \underline{-30000,} & = \frac{1}{3}rr \\ r + e = 366, & 24030,1388 & = D. \quad (66 = e) \\ & \underline{216} & = re + ee \\ & 2430 & \\ \text{New } r = 366 & \underline{2196} & = re + ee. \end{array}$$

But as was said above, I take but one place, then

$$\begin{array}{rcl} r = 360 & \text{and } 45025,1157 & = \frac{1}{3}G \div r \\ & \underline{-43200,} & = \frac{1}{3}rr \\ r + e = 365,0003 & 1825,1157 & = D. \\ & \underline{1825,} & = re + ee \quad (5,0003 = e) \\ & 0,11570000 & \\ \text{hence } a = 365,0003 & \underline{10150009,} & = re + ee \end{array}$$

The Root is really but 365, this overplus of 0003, would at the next Operation vanish. And from this Example, 'tis apparent, that Four Places will truly arise in the Root, at the Second Operation.

The work of these Two last Examples, (I presume) is so plain and clear, that it's needless to express the Process thereof in Words: Also from hence it's as plain, that the Extraction of the Cube Root, (heretofore so very difficult) is by this Method rendered easier, than that of the Square, as it's usually performed.

Extraction of Roots.

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III. Of Extracting the Biquadrate Root, viz.

$aaaa = G$. What is the value of a ?

Let $aaaa = G = 2741583974$, and $r + e = a$.

Then

1	$rrrr + 4rrre + 6rree + 4reee + eeee = G$
$1 \div 4$	2 $\frac{1}{4}rrrr + rrrr + 1\frac{1}{2}rree + reee + \frac{1}{4}eeee = \frac{1}{4}G$
$2 \div rr$	3 $\frac{1}{4}rr + re + 1\frac{1}{2}ee + \frac{eee}{r} + \frac{\frac{1}{4}eee}{rr} = \frac{1}{4}G \div rr$

Let $eee \div r$, and $\frac{1}{4}eeee \div rr$ be rejected, or cast off, (as before,) then it will become

$$\frac{1}{4}rr + re + 1\frac{1}{2}ee = \frac{1}{4}G \div rr$$

and consequently $re + 1\frac{1}{2}ee = \frac{\frac{1}{4}G}{rr} - \frac{1}{4}rr = D$.

From hence ariseth this

Theorem $\left\{ \frac{D}{r + 1\frac{1}{2}e} = e \right.$

First $r = 200$.

$$2741583974 = G$$

$$685395993,5 = \frac{1}{4}G$$

$$17134899 \text{ & c.} = \frac{1}{4}G \div rr$$

$$- 10000 = \frac{1}{4}rr$$

$$r + e = 229$$

$$+ \frac{1}{2}e = 14,5$$

Divisor 23

Divisor 233,5

$$713489 \text{ & c.} = D(29 = e$$

$$46 = re + 1\frac{1}{2}ee$$

$$2534,8$$

$$2101,5 = re + 1\frac{1}{2}ee.$$

New $r = 229$,

$$- 1\frac{1}{2}e = ,15$$

Divisor 228,85

New $r = 228,9$

$$- 1\frac{1}{2}e = ,105,$$

Divisor 228,795,

New $r = 228,83$

$$- 1\frac{1}{2}e = ,009,$$

Divisor 228,821

$$13069,8498 = \frac{1}{4}G \div rr$$

$$- 13110,25 = \frac{1}{4}rr$$

$$- 40,4002 = D(-,1 = e$$

$$22,885 = re - 1\frac{1}{2}ee$$

$$17,51520 (-7 = e$$

$$16,01565 = re - 1\frac{1}{2}e$$

$$1,499550 (6 = e$$

$$1,372926 = re - 1\frac{1}{2}e$$

$$1266240 (5 = e$$

$$1144105$$

$$r = 229,$$

$$- ,1765$$

$$a = 228,8235$$

1a.

56 Extraction of Roots.

In this Example, it appears I took the Second $r = 229$ too big, and therefore in the Second Process, the Member e , Convergeth with the Sign $-$ and consequently (in this Case) the Divisor, must be $re - 1\frac{1}{2}e$; also at each Division, e , is to be taken from the last r , and the remainder will be the New r .

IV. Of Extracting the Root of a Surfolide, (or Fifth Power,) viz. $aaaaa = G$. What's the value of a ?

In Numbers $aaaaa = 43467897 = G$.

Put	1	$r + e = a$. Reject all the Powers of e above ee .
then	2	$rrrrr + 5rrrre + 10rrree = G$
$2 \div \sqrt[5]{}$	3	$\frac{1}{3}rrrrr + rrrre + 2rrree = \frac{1}{3}G$
$3 \div r^3$	4	$\frac{1}{3}rr + re + 2ee = \frac{1}{3}G \div rrr$
$4 - \frac{1}{3}rr$	5	$re + 2ee = \frac{\frac{1}{3}G}{rrr} - \frac{1}{3}rr = D$

From hence ariseth this

$$\text{Theorem, } \sum \frac{D}{r + 2e} = e.$$

First $r = 30$

$$\begin{array}{r} 43467897 = G \\ 8693579.4 = \frac{1}{3}G \\ 321,96,000 = \frac{1}{3}G \div rrr \\ \underline{-180,000} = \frac{1}{3}rr \\ 141,960 = D \quad (3 = e) \\ \underline{108000} = re + 2ee \end{array}$$

$$\begin{array}{r} r + e = 33 \\ + e = 3 \end{array}$$

Divisor 36

$$\begin{array}{r} \text{New } r = 33, \\ + 2e = 1.4 \end{array}$$

Divisor 34.4

$$\begin{array}{r} \text{New } r + e = 33.7009 \\ + e = .0009 \end{array}$$

Divisor 33.7018

$$\begin{array}{r} 241,911,600.19 = \frac{1}{3}G \div r^3 \\ \underline{-217,800} = \frac{1}{3}rr \\ 24,111,600.19 = D, 7 = e \\ \underline{24,0800} = re + 2ee \\ .03160019 (.0009 = e) \\ \underline{3033162} = re + 2ee \end{array}$$

$$r + e = 33.7009 = a.$$

But if more Exactness be required, the next Operation will produce 17 places of Figures true.

I doubt not, but by these few Examples, the nature of, and manner how to proceed in, this Method is sufficiently cleared; as to the Extraction of Roots out of Simple or pure Equations, how highly soever they be.

And because there is great care and trouble attends the continued Involutions of a Binomial, (to wit of $r + e$) especially to any considerable height, by reason of the *Uncia*, (or Numeral Figures, that arise by involving the Quantities) I will here shew, how any Power, of what Degree soever, may be speedily and easily raised: without Involving the Quantities at all. Thus

First make the Index of the highest Power of r , the same to that of the given Power of a ; then the next Power of r , (into e), must have its Index less by Unity, and the Third Power of r (into ee) must have its index less than the First by 2, that is, their Indices decrease in Arithmetical Progression.

Next for obtaining the *Uncia*, or respective Numeral Figures that would arise by a continued Involution of $r + e$, prefix to the Second Power of r , (into e), the Index of the First (or simple) Power of r . And to the Third Power of r (into ee) prefix half the Product of the first Index, into the Index of the Second, and these will be the true Numbers required.

For the clearing of this, take an Example or Two. Suppose a^9 , were given, and you are to raise $r + e$ to the same height (that is, so much thereof as is required in this Method) first set down r^9 , to this add r^8e , with the Index of the first prefix'd to it; then it will be $r^9 + 9r^8e$. To these add r^7ee , with half the Product of 9×8 , to wit, $9 \times 4 = 36$, and then it will be $r^9 + 9r^8e + 36r^7ee = a^9$, as was required.

Again, suppose a^{18} , were given; to raise the rest as before, $r^{18} + 18r^{17}e$, are the Two first Members, and the half of 18×17 , that is, $9 \times 17 = 153$, and then they will be $r^{18} + 18r^{17}e + 153r^{16}ee = a^{18}$.

And this I take to be a new Discovery, having never seen any thing performed in this kind, with the like ease and expedition.

CHAP.

CHAP. X.

The Resolving of several Geometrical Problems, together with the Solution of their Equations in Numbers.

FOR the Resolving of *Geometrical Problems*, it will be requisite to have a clear understanding of these Two Propositions of *Euclid*, viz. The 47th of the 1st, and 4th of the 6th Book, of their General, (and excellent) use in *Geometry*, (and consequently in all parts of the *Mathematicks*;) none that hath been conversant therein, but will readily grant; of which *Des Cartes* takes particular notice, in a Letter of his, cited by *Dr. Pell*, in the before-mentioned Book of *Algebra*, Pag 65.

And here it may not be amiss (for the Benefit of the Young Learner,) to give a particular account of the afore-said Propositions.

Prop. I. In all Right-angled Triangles, the Square of the Hypotenuse (or greatest side) is equal to the Sum of the Squares of the Base, and Perpendicular, (or of the other Two sides.) *Eucl. 1. 47.*

Suppose the Triangle *CAB*, to be right-angled at *A*. Then let

BC = *b* = Hypotenuse,

AC = *p* = Perpendicular,

AB = *b* = Base.

And then it is

$$bh = bb + pp$$

$$bb = bh - pp$$

$$pp = bh - bb$$

That is, in Numbers thus; suppose

b = 5, *b* = 4, and

p = 3, as in the

Scheme. Then is,

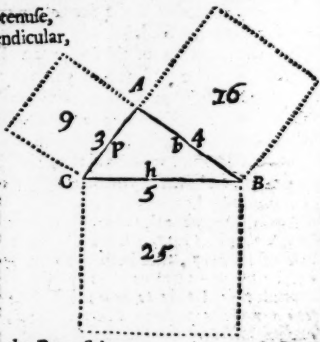
$$bh = 25, \text{ and}$$

$$bb = 16, \text{ and}$$

$$pp = 9, \text{ but}$$

$$16 + 9 = 25,$$

which makes good the Proposition.



Prop.

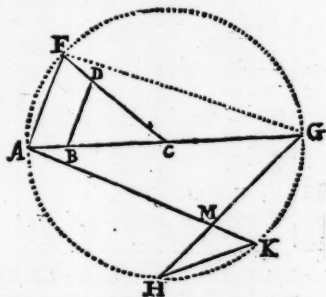
Of Geometrical Problems. 59

Prop. II. The sides of like Triangles, are respectively proportional to each other, *Eucl. 6. 4.*

That is, if there be Two Triangles, (either Right or Oblique) that have each Angle of the one, equal to each Angle of the other, then are those Two Triangles alike, and their respective sides proportional.

Angles are equal, that stand upon, (or have) equal Arches, either both from the Center, or both from the Periphery of a Circle. As for Example, Suppose the Two Triangles to be ACF , and BCD , having the Angle at C , (the Center) common to both, and BD , parrallel to AF .

Then is the Angle at D , equal to that at F , and consequently the Angle at B , equal to that at A . And according to the Proposition, $AC:AF::BC:BD$. and $FC:AC::DC:BC$. &c.



Again the Triangles AMG , and HMK are alike; for the Angles A , and H , stand both upon the Arch GK , and the Angles G , and K , stand both upon AH ; also the Angle at M , is alike in both. Therefore according to the Proposition,

$AG:HK::AM:HM$, and $GM:MK::AM:MH$, &c.

To these Two, add, that the Angle at the Center, is double to that at the Periphery; that is, the Angle FGA , is Double to the Angle FGA . *Eucl. 3. 20.*

These

60 Resolving of Problems,

These Propositions being premised, and pretty well understood, the young Algebrist may proceed to the Resolution of Problems, into which I endeavour to introduce him, with some that be easie.

PROB. I.

IN the Right Angle Triangle BAC , there is given the Sum of the Sides AB , and AC ; also the Perpendicular P , let fall from the Right Angle A , upon the Hypotenuse, BC . To find the Hypotenuse, and the Sides.

Let the Hypot. : $BC = a$, $AC = e$. $AB = y$.

Then	1	$e + y = z$	} given { $z = 48$ $p = 17$
	2	$p = p$	
	3	$BC = a$, sought	
Prop. 1.	4	$ee + yy = aa$	
Prop. 2.	5	$y : a :: p : e$	
5 ∴	6	$ye = pa$	
1 ⊙ 2	7	$ee + 2ye + yy = zz$	
6 × 2	8	$2ye = 2pa$	
4 + 8	9	$ee + 2ye + yy = aa + 2pa$	
7, 9,	10	$aa + 2pa = zz$	

This is called the First Form of Square Adjoined Equations, and may be solved, by completing the Square; (as in Page 31.) Then it will become

$$aa + 2pa + pp = zz + pp$$

and consequently $a + p = \sqrt{zz + pp}$.

Then will $a = \sqrt{zz + pp} - p$.

And this Method is easie enough, especially, by the Method of Extracting the Square Root. (Page 52.) But the same may be readier done by the following Method.

Notes

and their Equations. 61

Note, When Prop 1, or Prop. 2. is placed in the Margin, they refer to the Two Propositions before mentioned. Page 58, and 59.

The New found Equation is $aa + 2pa = xx$.

In Numbers $aa + 34a = 2304 = G$.

Put,	1	$r + e = a$
1 \odot 2	2	$rr + 2re + ee = aa$
1 \times 2p	3	$2pr + 2pe = 2pa$
3 + 2	4	$rr + 2pr + 2re + 2pe + ee = G$
4 \div 2	5	$\frac{1}{2}rr + pr + re + pe + \frac{1}{2}ee = \frac{1}{2}G$
Then	6	$re + pe + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr - pr = D$

From hence ariseth this

Theorem $\left\{ \frac{D}{r + p + \frac{1}{2}e} = a \right.$

$r = 30$ ~~less~~ $2304 = G$ and $17 = p$
 $\left. \begin{array}{l} rr = 450 \\ pr = 510 \end{array} \right\} = - \begin{array}{l} 1152 = \frac{1}{2}G \\ 960 = pr + \frac{1}{2}rr \end{array}$

$r + p = 47$ $192,00 = D$ (3. = e)
 $+ \frac{1}{2}e = 1,5$ $145,5$

Divisor $48,5$ $46,5000$ (62 = e)
 $45,405$

New $r + p = 50,$ $1,0950$ (2 = e)
 $+ \frac{1}{2}e = ,45$ $1,0182$

2 Divisor $50,45$ 768000 (1 = e)
 509205

New $r + p = 50,9$ 258795 (50823 = e)
 $+ \frac{1}{2}e = ,01$ 254605

Divisor $50,91$ 4190 For finding the
 4072 first r .

New $r + p = 50,92$ 118 Suppose $r = 4$
 $+ \frac{1}{2}e = ,0005$ 100

Divisor $50,9205$ 18 $rr = 16$
 15 $3r = 12$

New $r + p = 50,921$ 15 $28 > 23$
 15 Ergo, $r < 4$

* Here I desist forming a new Divisor 15 Or

First $r = 30$ $3,92150823$

$+ e = 3,92150823$ This is also a continued Series

the Square Root was.

G PROB.



62 Resolving of Problems,

P R O B. II.

Suppose in the Triangle BAC ; there were given d = the difference of the Sides AB , and AC . Also the Perpendicular P , let fall from the Right Angle A , upon the Hypotenuse BC . Thence to find the Hypotenuse, &c.

Let $BC = a$. $AC = e$. And $AB = y$. (as before.)

$$\text{Then } \left\{ \begin{array}{l} 1 \quad y - e = d \\ 2 \quad p = p \\ 3 \quad BC = a \text{ sought.} \end{array} \right\} \text{ given } \left\{ \begin{array}{l} d = 14 \\ p = 20. \end{array} \right.$$

$$\text{Prop. 1. } 4 \quad yy + ee = aa$$

$$\text{Prop. 2. } 5 \quad y : a :: p : e$$

$$5 \cdot 6 \quad ye = pa$$

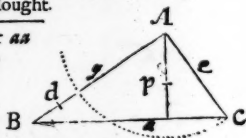
$$6 \times 2 \quad 7 \quad 2ye = 2pa$$

$$1 \text{ } \textcircled{+} 2 \quad 8 \quad yy - 2ye + ee = dd$$

$$8 + 7 \quad 9 \quad yy + ee = dd + 2pa$$

$$A, 9, \quad 10 \quad aa = dd + 2pa$$

$$\text{Then } 11 \quad aa - 2pa = dd.$$



This is called the Second form of Square Adaffected Equations.

And may be solved by Compleating the Square, as in the first Form, respect had to the Sign —

Then it will become

$$aa - 2pa + pp = dd + pp.$$

$$\text{And consequently, } a - p = \sqrt{dd + pp}.$$

$$\text{Then will } a = \sqrt{dd + pp} + p.$$

But this Form may be solved (by my Method) as was the last, to wit, by a continued Series.

Equation

and their Equations.

63

Equation $aa - 2pa = dd.$

In Numbers, $aa - 40a = 196 = G,$

Put	1	$r + e = a$
1 \odot 2	2	$rr + 2re + ee = aa$
1 \times 2p	3	$2pr + 2pe = 2pa$
2 - 3	4	$rr + 2re - 2pr - 2pe + ee = G$
4 \div 2	5	$\frac{1}{2}rr + re - pr - pe + \frac{1}{2}ee = \frac{1}{2}G$
Then	6	$re - pe + \frac{1}{2}ee = \frac{1}{2}G + pr - \frac{1}{2}rr = D$

From hence ariseth this

$$\text{Theorem } \left\{ \frac{D}{r - p + \frac{1}{2}e} = e. \right.$$

$r > 30$, because of $-2pr$. Suppose $r = 30$. Then $rr = 900$, and $2pr = 1200$. But 1200 cannot be taken out of 900 , therefore $r > 30$. Suppose $r = 50$, $rr = 2500$, and $2pr = 2000$, then $rr - 2pr = 500$, too much, Ergo $r > 30$, but $r < 50$.

Let $r = 40$	$196 = G$	and $20 = p$
$+ pr = 800$	$98 = \frac{1}{2}G$	
$- \frac{1}{2}rr = 800$	$00 = pr - \frac{1}{2}rr$	
$r - p = 20$	$\frac{98}{88} = D$	(4 = e
$+ \frac{1}{2}e = 2$	$88,$	
Divisor 22,	10,00	(,4 = e
$r - p = 24$	9,68	
$+ \frac{1}{2}e = 2$	32000	(1 = e
$r - p = 24,4$	24405	
$+ \frac{1}{2}e = ,005$	759500	(3 = e
$r - p = 24,41$	732345	
$+ \frac{1}{2}e = ,0015$	* 27155	(11124 = e
$r - p = 24,413$	24413	

*Here I desist forming a new Divisor.

First $r = 40$	
$+ e = 4,41311124$	57
$a = 44,41311124$	48

Qc.

PROB.

64 Resolving of Problems,

P R O B. III.

IN the Right Angle Triangle BCA , the Base (or Side) BA , is given, And the Hypotenuse with the Area, in one Sum; thence to find the Perpendicular AC .

Let $AC = a$, $BC = e$, and $BA = b$, Given.

Then	1	$\frac{1}{2}ba = \text{the Area}$	the } Sum of the Hypot. and Area.
And	2	$e + \frac{1}{2}ba = z$	
Prop. 1.	3	$ce = bb + aa$	
$2 - \frac{1}{2}ba$	4	$e = z - \frac{1}{2}ba$	
$4 \odot 2$	5	$ee = zz - zba + \frac{1}{4}bbaa$	
$3, 5,$	6	$bb + aa = zz - zba + \frac{1}{4}bbaa$	
$6 \pm$	7	$aa - \frac{1}{4}bbaa + zba = zz - bb$	

Let $z = 69$, and $b = 7$, then the Equation in Numbers will be $42,93a - aa = 418,844.4$ &c.

This is a Square Affected Equation of the Third Form.

Let it be made $ba - aa = G$.

This Form used to be Resolved, by completing the Square (as before) but first the Equation is supposed to be changed, and made

$$aa - ba = -G.$$

Then $aa - ba + \frac{1}{4}bb = \frac{1}{4}bb - G$.

And $a - \frac{1}{2}b = \sqrt{\frac{1}{4}bb - G}$. Hence, $a = \frac{1}{2}b \pm \sqrt{\frac{1}{4}bb - G}$.

But I shall solve this by my Method,

Thus,	1	$r + e = a$
$1 \odot 2$	2	$rr + 2re + ee = aa$
$1 \times b$	3	$br + be = ba$
$3 - 2$	4	$br - rr + be - 2re - ee = ba - aa = G$
$4 \div 2$	5	$\frac{1}{2}br - \frac{1}{2}rr + \frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G$

Then will $\frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G + \frac{1}{2}rr - \frac{1}{2}br = D$.

From

and their Equations.

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From hence ariseth this

$$\text{Theorem } \left\{ \frac{D}{\frac{1}{2}b - r - \frac{1}{2}e} = e. \right.$$

$$42,93 = b$$

$$418,8444 = G.$$

If $r = 20$, then $rr = 400$, and $br = 858,6$

But $858,6 - 400 = 458,6$ Ergo, $r < 20$.

Let $r = 10$

$$+ \frac{1}{2}rr = 50$$

$$418,8444 = G$$

$$- \frac{1}{2}br = 214,65$$

$$209,4222 = \frac{1}{2}G$$

$$- 164,65 = 164,65 = \frac{1}{2}rr - \frac{1}{2}br$$

$$\frac{1}{2}b - r = 11,465$$

$$44,7722 = D$$

$$(4 = e$$

$$- \frac{1}{2}e = 2$$

$$37,860$$

$$\text{Divisor } 9,465$$

$$6,9122$$

$$(9 = e$$

$$\text{New } r = 14$$

$$6,3135$$

$$(9 = e$$

$$\frac{1}{2}b - r = 7,465$$

$$59870$$

$$- \frac{1}{2}e = ,45$$

$$58680$$

$$\text{Divisor } 7,015$$

$$119000$$

$$(1 = e$$

$$\text{New } r = 14,9$$

$$64745$$

$$(838 = e$$

$$\frac{1}{2}b - r = 6,565$$

$$542550$$

$$- \frac{1}{2}e = ,045$$

$$517960$$

$$\text{Divisor } 6,520$$

$$24590$$

$$\text{New } r = 14,99$$

$$19422$$

$$\frac{1}{2}b - r = 6,475$$

$$5168$$

$$- \frac{1}{2}e = ,0005$$

$$5176$$

$$\text{Divisor } 6,4745$$

$$Oc.$$

• Here I desist forming

a new Divisor.

First $r = 10$,

$$+ e 4,991838$$

$$= 14,991838 Oc.$$

This Third Form is something more troublesome than they Two first; but a little practice will render it very easy. And this is also a continued Series, (as before.)

G 3

PROB.

D

66 Resolving of Problems,

PROB. IV.

IN the Oblique Triangle BCD , there is given the Base (or Side) BD , with the Side CD ; and if the Difference of the Sides BC , and CD , to wit, $BC - CD$, be Multiplied into the Side CD , that Product must be equal to the Square of the Segment of the Base, *viz.* $= aa$.

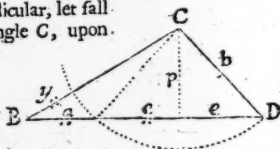
By these to find out the Segment, a .

Let $\left\{ \begin{array}{l} 1 \mid BD = d \\ 2 \mid CD = b \end{array} \right\}$ given $\left\{ \begin{array}{l} d = 92 \\ b = 56 \end{array} \right.$
 And $3 \mid BC - DC = y$
 Then $4 \mid by = aa$, by the Problem.

Suppose a Perpendicular, let fall from the Obtuse Angle C , upon the Side BD , call it p .

Also let $\frac{d-a}{2} = e$.

as in the Scheme.



Then

Prop $\left\{ \begin{array}{l} 5 \mid pp + ee = bb \\ 6 \mid aa + 2ae + ee + pp = bb + 2by + yy \\ 6 - 5 \mid 7 \mid aa + 2ae = 2by + yy. \end{array} \right.$
 That is, $8 \mid a + 2e : 2b + y :: y : a$.

And this proves Proposition, 35, 36, 37. *Eucl.* 3.

To wit,

$8, \therefore 9 \mid 2by + yy = da$, for $d = a + 2e$.

$4 \times 2 \mid 10 \mid 2by = 2aa$

$4 \odot 2 \mid 11 \mid bbyy = aaaa$

$11 \div bb \mid 12 \mid yy = \frac{aaaa}{bb}$

$10 + 12 \mid 13 \mid 2by + yy = 2aa + \frac{aaaa}{bb} = da$

$13 \times bb \mid 14 \mid 2bbaa + aaaa = bbda$

$14 \div a \mid 15 \mid aaa + 2bba = bbd$

and their Equations. 67

In Numbers, $aaa + 6272a = 288512 = G$.

Let it be made $aaa + ba = G$.

Put	1	$r + e = a$
1 \odot 3	2	$rrr + 3rre + 3ree = aaa$
1 $\times b$	3	$br + be = ba$
2 $+ 3$	4	$r^3 + br + 3rre + be + 3ree = G$
4 $\div 3$	5	$\frac{1}{3}r^3 + \frac{1}{3}br + rre + \frac{1}{3}be + ree = \frac{1}{3}G$
5 $\div r$	6	$\frac{1}{3}rr + \frac{1}{3}b + re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r}$
6 $-$	7	$re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r} - \frac{1}{3}rr - \frac{1}{3}b = D$

From hence ariseth this

$$\text{Theorem } \left\{ \frac{D}{r + \frac{\frac{1}{3}b}{r} + e} = e \right.$$

$$6272 = b \quad 288512 = G$$

If $r = 40$, $rrr = 64000$, $br = 250880$.

But $250880 + 64000 = 314880$. Ergo $r < 40$.

Let $r = 30$

$$\begin{array}{rcl} \frac{1}{3}rr & = & 300 \\ \frac{1}{3}b & = & 2090,6 \\ - 2390,6 & = & 2390,666 = \frac{1}{3}rr + \frac{1}{3}b \end{array} \quad \begin{array}{rcl} 56170,66 & = & \frac{1}{3}G \\ 3205,688 & = & \frac{1}{3}G \div r \end{array}$$

$$\frac{\frac{1}{3}b}{r} + r = 99,68 \quad 815,022 = D$$

$$\begin{array}{r} + e = 7 \\ \text{Divisor } 106, \quad 742 \end{array} \quad (7 = e)$$

$$\begin{array}{rcl} \text{New } r = 37, & 2599,207207 & = \frac{1}{3}G \div r \\ \frac{1}{3}b \div r = 56,504 & - 2546,999999 & = \frac{1}{3}rr + \frac{1}{3}b \end{array}$$

$$\frac{\frac{1}{3}b}{r} + r = 93,504 \quad 52,207208 = D$$

$$\begin{array}{r} + e = ,5 \\ \text{Divisor } 94,0 \quad 47,00 \end{array} \quad (,5 = e)$$

$$\begin{array}{r} + e = ,05 \\ \text{Divisor } 94,05 \quad 47025 \end{array} \quad (5 = e)$$

$$\begin{array}{r} + e = ,005 \\ \text{Divisor } 94,059 \quad 470295 \end{array} \quad (5 = e)$$

$$\begin{array}{r} \text{Last } r = 37, \\ + e = ,5553 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} = 37,5553 = \text{PROB.}$$



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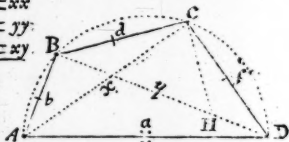
P R O B. V.

THE Three Chords (or Subtences) of three Archies, completing a Semicircle, being each given: Thence to find the Diameter of that Circle.

Suppose, $AB = 3 = b$, $BC = 4 = d$, and $DC = 5 = f$.

To find $AD = a$, the Diameter, Draw the Two Diagonals, $BD = y$, and $AC = x$. Then is

$$\begin{array}{l} \text{Pro. 1} \left\{ \begin{array}{l} 1 \quad aa - ff = xx \\ 2 \quad aa - bb = yy \\ \text{And } 3 \quad da + bf = xy \end{array} \right. \end{array}$$



This Third Step I have often heard asserted, but never could see it clearly demonstrated, which I undertake to do in this manner.

Make CH Perpendicular to BC , then the Triangles ACD , and BCH , are alike, both Right angled at C , and having the Angles at A and B , both measured by the Arch CD . Again, the Triangles ABC , and CHD are alike; for the Angles at A , and D , stand both upon the Arch BC , and the Angles DCH , and BOA are equal; For the Angles $BCA + ACH = 90$ Degrees, also the Angles $DCH + ACH = 90$ Degrees. That is, the Angles $DCH + ACH = BCA + ACH$; and consequently the Angle $DCH = BCA$; these being proved, it will be $AC : AB :: DC : DH$, and $AC : AD :: BC : BH$.

That is, 4 $x : b :: f : DH$, and $x : a :: d : BH$

4 \therefore 5 $\frac{bf}{x} = DH$, and $\frac{da}{x} = BH$, but $y = DH + BH$

therefore 6 $\frac{bf}{x} + \frac{da}{x} = y$. Hence $bf + da = xy$.

1 \times 2. 7 $aaaa - aaff - bbaa + fffb = xxyy$

3 \odot 2 8 $ddaa + 2dbfa + bbff = xxyy$

7, 8, 9 $aaaa - ffaa - bbaa = ddaa + 2dbfa$

9 \div 8 10 $aaa - ffa - bba - dda = 2dbf$

Which in Numbers will be, $aaa - 50a = 120$.

Let

and their Equations. 69

Let it be made $aaa - ba = G$. This is an Equation of the Second Form of Cubicks, and may be solved with the same Theorem of the last, only changing the Sign of b .

Then it will be $re - \frac{1}{3}b + ee = \frac{1}{3}G + \frac{1}{3}b - \frac{1}{3}rr = D$

From hence arifeth this

$$\text{Theorem } \left\{ \frac{D}{r - \frac{1}{3}b + e} = e \right.$$

Let $r = 6$, then $rrr = 216$, and $br = 300 > 216$
Ergo $r > 6$.

Suppose $r = 8$, $rrr = 512$, and $br = 400$, then $512 - 400 = 112$, hence it appears that $r > 8$, but not 9.

Let $r = 8$

$$\begin{array}{rcl} 120 & = & G \\ 40 & = & \frac{1}{3}G \\ \frac{1}{3}rr & = & 21,3333 \\ + \frac{1}{3}b & = & 16,6666 \\ - 4,6667 & = & 4,6667 = \frac{1}{3}b - \frac{1}{3}rr \\ \frac{1}{3}b \div r & = & 2,083 \\ r - \frac{1}{3}b & = & 5,917 \\ + e & = & ,055 \\ 1 \text{ Divisor } & 5,96 & \\ 2 \text{ Divisor } & 5,972 & \end{array}$$

New $r = 8,955$

$$\begin{array}{rcl} - \frac{1}{3}rr & = & 21,627675 \dots 40 = \frac{1}{3}G \\ + \frac{1}{3}b & = & 16,66666666 \dots 4,9658597144 = \frac{1}{3}G \div r \\ - 4,9610083 & = & 4,9610083333 \end{array}$$

$$\begin{array}{rcl} r - \frac{1}{3}b & = & 5,9859 \\ + e & = & ,0008 \\ \text{Divisor } & 5,9867 & \\ + e & = & ,00001 \\ \text{Divisor } & 5,98671 & \end{array}$$

$$\begin{array}{rcl} \text{Divisor } & 5,98671 & \\ * & 2154000 & \\ & 1796013 & (0359 = e \\ & 3579980 & \\ & 2993355 & \\ & 5866250 & \\ & 5389039 & \end{array}$$

*Here I desist increasing the Divisor with e .

$$\begin{array}{rcl} \text{Last } r & = & 8,055 \\ + e & = & ,000810359 \\ e & = & 8,055810359 \end{array}$$

PROB.

and their Equations.

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Let it be made $ba - aaa = G.$

Put	1	$r + e = a$
1 \times 3	2	$r^3 + 3rre + 3ree = aaa$
1 \times b	3	$br + be = ba$
3 - 2	4	$br - r^3 + be - 3rre - 3ree = G$
2 \div 3	5	$\frac{1}{3}br - \frac{1}{3}rrr + \frac{1}{3}be - rre - ree = \frac{1}{3}G$
5 \div r	6	$\frac{1}{3}b - \frac{1}{3}rr + \frac{\frac{1}{3}b}{r} - re - ee = \frac{1}{3}G \div r$
6 \div	7	$\frac{\frac{1}{3}b}{r} - re - ee = \frac{\frac{1}{3}G}{r} + \frac{1}{3}rr - \frac{1}{3}b = D.$

From hence arifeth this

Theorem $\left\{ \frac{\frac{1}{3}b}{r} - r - e = e. \right.$

Being $c = 1.$ $r > ,3$ but $r < ,4.$

Let $r = ,3$ $1 = G$ $b = 3$
 $\frac{1}{3}rr = ,03$ $,333333 = \frac{1}{3}G$
 $-\frac{1}{3}b = 1,00$ $1,111111 = \frac{1}{3}G \div r$
 $-,97 = ,97 = \frac{1}{3}rr - \frac{1}{3}b$
 $\frac{1}{3}b \div r = 3,3333$ $,141111 = D$ ($,04 = e$)
 $-r = ,3$ 119732
 $\frac{1}{3}b - r = 3,0333$ 213790 ($7 = e$)
 $-e = ,047$ 209041

Divisor $2,9933$
 Divisor $2,9863$

New $r = ,347$ $,960614793458 = \frac{1}{3}G \div r$
 $\frac{1}{3}rr - \frac{1}{3}b = - ,959863666667$

$\frac{1}{3}b - r = 2,534844$ $,000751126791 = D$
 $-e = ,0002$ 50692 ($,0002 = e$)
 Divisor $2,5346$ 2442067 ($9 = e$)
 $-e = ,00009$ 2281095
 Divisor $2,53455$ 16097291 ($6 = e$)
 $-e = ,000006$ 15207288
 Divisor $2,534548$ 8900030 ($35 = e$)

Last $r = ,347$ 7603644
 $+e = ,00029635$ 1296386
 $= ,34729635$ 1267270 In

De

72 Resolving of Problems,

In the preceding Examples, I have inserted such Problems, as would produce the Three Forms (as they are usually called) of Square, and Cubick Adfectèd *Æquations*, (each in it's Order,) that is, such as consist of the highest Power thereof, mixt with its Root.

How other *Æquations* that are Mixt, or Adfectèd with their intermediate Terms, (or Inferiour Powers) to wit,

$$aaa + baa - ca = d, \text{ or } baa - ca - aaa = d, \&c.$$

may be reduced to some of the Three foregoing Forms, might here be shewed, by those commonly called *Cardan's Rules*, or otherwise; but such Proceſs are too large and tedious, to be inserted in this small Tract: Neither (in truth) have we any need of such Expedients, as the casting off the Second Term of an *Æquation*, &c. which was invented purely to render the same more fit and easie for a Solution in Numbers. For by the Methods of Converging Series, any *Æquation* may be as well solved, (still keeping its own Form, as the Problem produced it,) as those of the Three Forms.

I have also omitted the Doctrin of *Surds* for the like Reason, to wit, because it requires a large Explanation to render it intelligible, without which, it's very intricate and troublesome, and rather puzzles or confounds a Learners Genius, than improves it, neither could I ever (yet) perceive any great Use or Advantage thereby, in Resolving any Problem, (and afterwards we have now no occasion for it.) For admit a Problem (or any part thereof) were proposed in *Surds*, it's much easier (in my Opinion) to bring it out of such *Surds*, before they be engaged in any Proceſs, than after they are become intangled, and mixt with other Quantities concern'd therein.

How such Work may be performed, is already shewed.

I could likewise have inserted Rules for discovering of the several Roots of an Adfectèd *Æquation*, whether Affirmative or Negative, &c.

But

and their Equations.

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But I content my self with one Root only, that is, such a one as suits or agrees best with the Nature and Design of the Problem proposed, Leaving those Speculations to be inquired after (by such as desire them) in the Works of *Des Cartes*, or Dr. *Wallis's* Treatise of *Algebra*, wherein they may meet with ample satisfaction in each particular case, &c.

Before I proceed to the Resolving of other kinds of Affected Equations, it may be convenient to say something of those that are already done.

Perhaps it may be suggested by some, that this Method requires too many Operations in Division, as Two, Three or sometimes more, according to the Number of Terms in the Equation.

That it doth require several Divisions is true, but without consider, that those Divisions are performed with small Divisors, and that each Operation is in order to bring down both the Resolvend and Divisor into lower Terms, which was the very thing I first aimed at.

However, to accommodate such as prefer *Multiplication* (though the Factors be large) before *Division*, (notwithstanding the Divisors be small,) I will here insert a different Method of ordering the several Powers of the Root r , with the Coefficients, &c. which perhaps may seem easier to some, and therefore may please better; tho both come to the same in the conclusion.

And since I have entered upon the Subject of Angular Sections, I will proceed a little further therein; and Resolve some of the odd Sections, viz. The Fifth, Seven and Ninth, passing over those of the Even Powers, to wit, the Fourth, Sixth and Eight, because they may be Resolved by that of the Bisection only; for that Reason it will be convenient (though out of its due course) to insert the Bisection of an Angle.

H

P R O B.

and their Equations.

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Consequently, $\left\{ \frac{D}{1,4 + e} = e \right.$

$$\begin{array}{r} + e = 1,4 \\ \text{Divisor } 1,4176 \end{array}$$

$$\begin{array}{r} ,0250 = D \\ 141 \quad (,0176 = e \\ \hline 10900 \\ 9919 \\ \hline 98100 \\ 85056 \end{array}$$

First $r = ,5$
 $+ e = ,0176$
 New $r = ,5176$

I take but $,517 = r$, which being involved as before, the Numbers will be

$$\begin{array}{r} - ,0714434 - ,55275e - 1,6037ee \\ + 1,0691560 + 4,13600e + 4,00000e \end{array}$$

That is $,9977126 + 3,58425e + 2,3963ee = 1$
 Then $3,58425e + 2,3963ee = ,0022874$
 And $1,49593e + ee = ,0009547120 = D$

Consequently, $\left\{ \frac{D}{1,49593 + e} = e \right.$

$$\begin{array}{r} 1,49593 \quad ,0009547120 = D \\ + e = ,0006 \quad 89790 \quad (,0006 = e \\ \text{Divisor } 1,4965 \quad 568120 \\ + e = ,000037 \quad 448968 \quad (37 = e \\ \text{Divisor } 1,496567 \quad 11915200 \\ \quad 10475969 \end{array}$$

*Here I defist increasing the Divisor.

$$\begin{array}{r} \bullet 14392310 \quad (9616 = e \\ 13469103 \\ \hline 923207 \\ 897936 \\ \hline 25271 \\ 14965 \\ \hline 10306 \\ \text{Oe.} \end{array}$$

Last $r = ,517$
 $+ e = ,0006379616$
 $\bullet = ,5176379616$

22,3176381041 No.

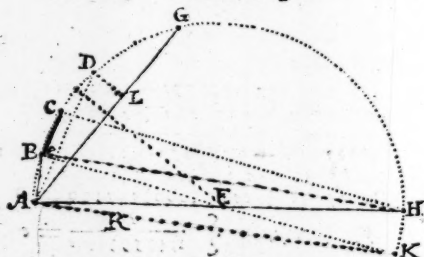
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PROB.

PROB. VIII.

THE Radius of a Circle, and the Chord of an Arch (suppose of 60 gr. as before) being given, to find the Chord of one fifth part of that Arch (to wit, of 12 gr.)

Let $AE = R$, Radius, } given { $R = 1$ }
and $AG = c$, Chord, } $c = 1$ }
 $AB = a$, the Chord sought.



By the Third Step in the Process of the Trisection of an Angle, *Pag.* 70, it was found that $\frac{aa}{R} = c$ and by

the Tenth Step, $\frac{3RRa - aas}{RR} = AD$

Make DL , perpendicular to AD .

Then are the Triangles AHC , and DAL , alike:

And $AH:CH::AD:AL$. But $AH = 2R$

But $CH = BK = c$ that is, $2R - R$

Fig.

Again, $AD = \frac{3RRa - aas}{RR}$, from hence it will be

$$2R:2R - \frac{aa}{R} :: \frac{3RRa - aas}{RR} : AL$$

$$\text{That is, } \frac{6R^4a - 5RRaas + a^5}{2RRRR} = AL$$

But

and their Equations.

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From E Draw a perpendicular on AG, intersecting AG in CD.

But $AL = \frac{1}{2}AG + \frac{1}{2}CD$, and $\frac{1}{2}CD = \frac{1}{2}a$

therefore, $\frac{6R^2a - 5RRaaa + aaaaa}{2RRRR} = \frac{1}{2}c + \frac{1}{2}a$,

That is, $aaaaa - 5RRaaa + 5RRRRa = RRRRe$. 279.

In Numbers $aaaaa - 5aaa + 5a = 1$

Put $r + e = a$

Then $\begin{cases} + rrrrr + 5rrrre + 10rrree = aaaaa \\ - 5rrr - 15rre - 15ree = -5aaa \\ + 5r + 5e \dots \dots \dots = 5a \end{cases}$

If $c = 1$. $r > 2$ but $r < 3$ by the Scheme,

Then $\begin{cases} + ,00032 + ,008e + ,08ee = aaaaa \\ - ,04 - ,600e - 3,00ee = -5aaa \\ + 1,0 + 5,0e \dots \dots \dots = 5a \end{cases}$

That is, $+ ,96032 + 4,408e - 2,92ee = 1$

Then $4,408e - 2,92ee = ,03968$

And $1,5e - ee = ,01356763 = D$

Consequently, $\left\{ \frac{D}{1,5 - e} = e \right.$

$-e = \frac{1,5}{,009} \quad ,01356763 = D.$
 Divisor $1,491 \quad \frac{13419}{14863} \quad (,009 = e$

First $r = ,2$
 $+ e = ,009$

New $r = ,209$ which being ordered as before,

Then $\begin{cases} + ,0003987782 + ,00954e + ,09129ee \\ - ,0456466450 - ,65521e - 3,13500ee \\ + 1,045 + 5,0e \end{cases}$

That is, $,9997521332 + 4,35433e - 3,0437067ee$

Then $4,35433e - 3,0437ee = ,0002478668$

Each part divided by 3,0437 then it will become

$1,430604e - ee = ,000081436015 = D$

[H 3

Corr.

78. Resolving of Problems,

Consequently $\left\{ \frac{D}{1,430604 - e} = e \right.$

$1,430604)$	$,000081436015 = D$
$- e = ,00005$	$715275 \quad (,00005 = e$
Divisor $1,43055$	$9908515 \quad (6 = e$
$- e = ,000006$	8583288
Divisor $1,430548$	$132522700 \quad (9 = e$
$- e = ,0000009$	128749239
Divisor $1,4305471$	$37734610 \quad (263 = e$
	28610942
	9123668
	8583282
	540386
	429162
	Otc.
Last $r = ,209$	
$+ e = ,0000569263$	
$= ,2090569263$	

I have here, (according to promise, Page 73.) presented you with another Method of Resolving *Equations*, very different from the first; For in that the Members of the Solution (to wit, $r + e$) together with the Coefficients are so ordered, that from them (still remaining in their Species) there ariseth a Theorem for the Solution of the *Equation*.

But in this Method, it is otherwise; for when the Members $r + e$, are involved according to the *Equation*, and Multiplied into their respective Coefficients, then are they to be brought into Numbers, and a due Collection made thereof, according to their Signs and Places.

Next the Absolute Numbers therein (unmix'd with e) are to be compared with the Resolvend, and Added to, or Subtracted therefrom as their Sign denotes; after which, each part is to be divided by the Factor found, prefixed to ee , that so the same may be cleared: Consequently, a Divisor will arise, which must be either increased or lessened by the Converging e , according to its Sign, the which, I *presume*, doth plainly appear, in the Solution of this and the last *Equation*.

I shall give an Example or Two more, that so the Young Algebrist may the better judge of the difference betwixt this and the other, and practice which he fancies.

Note, That this Method doth triple the Number of Places in the Root, at each Operation, as the other doth.

P R Q B.

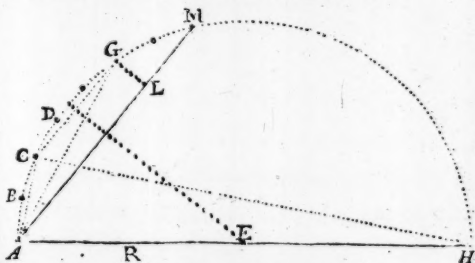
and their Equations.

79

P R O B. IX.

THE Radius of a Circle, and the Chord of an Arch being given, to find the Chord of the Seventh part of that Arch.

Let $AE = R$ Radius, $\left\{ \begin{array}{l} \text{given} \\ R = 1 \end{array} \right.$
and $AM = c$ Chord, $\left\{ \begin{array}{l} \text{given} \\ c = 1, \text{ the Chord of } 60^\circ \end{array} \right.$
Then $AB = a$, the Chord of $8 \text{ Degr. } 34' 19''$.



By the last Problem, it was found that

$$AG = \frac{aaaaa - 5RRaaa + 5RRRRa}{RRRR} = c. \text{ P. 77.}$$

And by that in Page 70, it was found that

$$\frac{3RRa - aaa}{RR} = AD. = c. \text{ P. 70.} \quad \text{And } CH = \frac{2RR - aa}{R} = \text{P. 76}$$

Make GL , perpendicular to AM , then are they Triangles AHC , and GAL alike; and therefore it will be

$AH:CH::AG:AL$. But $AH = 2R$
And $AL = \frac{1}{2}AM + \frac{1}{2}CG$, but $\frac{1}{2}CG = \frac{1}{2}AD$
then by taking those that are equal, it will be

$$2R: \frac{2RR - aa}{R} :: \frac{aaaa - 5RRa + 5R^2a}{RRRR} + \frac{3RRa - aaa}{2RR} \text{ P. 76}$$

If $R = 1$ as before, then it will become

$$2: 2 - aa :: a^5 - 5aaa + 5a: \frac{1}{2}c + \frac{3a - aaa}{2}$$

$$\text{Then } c + 3a - aaa = -a^5 + 7a^3 - 15a^2 + 10a$$

$$\text{That is, } -a^5 + 7a^3 - 14aaa + 7a = 1 = c \text{ Prob}$$

80 Resolving of Problems,

Put $r + e = a$. Then it will be

$$\begin{aligned} -r^7 - 7r^6e - 21r^5ee &= -a^7 \\ +7r^5 + 35r^4e + 70r^3ee &= +7a^5 \\ -14r^3 - 42rre - 42ree &= -14a^3 \\ +7r + 7e &= +7a \end{aligned} \quad \left. \vphantom{\begin{aligned} -r^7 - 7r^6e - 21r^5ee &= -a^7 \\ +7r^5 + 35r^4e + 70r^3ee &= +7a^5 \\ -14r^3 - 42rre - 42ree &= -14a^3 \\ +7r + 7e &= +7a \end{aligned}} \right\} = e$$

If $e = 1$ then $r > ,1$ but $r < ,2$ by the Scheme.

$$\begin{array}{r} \text{Then } \left\{ \begin{array}{l} - ,0000001 - ,000007e - ,00021ee \\ + ,00007 + ,0035e + ,07ee \\ - ,014 - ,42e - 4,2ee \\ + ,7 + 7,0e \end{array} \right. \end{array}$$

That is, $+ ,6860699 + 6,583493e - 4,13021ee = 1$

Then $6,583493e - 4,13021ee = ,3139301$

And $1,593e - ee = ,076008 = D$

$$\begin{array}{r} \text{Consequently } \left\{ \begin{array}{l} \frac{D}{1,539 - e} = e \\ -e = ,04 \\ \text{Divisor } 1,55 \end{array} \right. \quad \begin{array}{l} ,076008 = D \\ ,629 \\ 14008 \\ 13896 \end{array} \quad (,049 = e) \end{array}$$

$$\begin{aligned} \text{Last } r &= ,1 \text{ to which add } ,049 \text{ New } r = ,149 \\ - ,00000163044 - ,0000766e - ,001542ee \\ + ,00051407839 + ,0172508e + ,231566ee \\ - ,046311286 - ,932442e - 6,258ee \\ + 1,043 + 7,0e \end{aligned}$$

$+ ,99720116195 + 6,0847322e - 6,027976ee = 1$

That is, $1,0094137e - ee = ,000464340511 = D$

$$\begin{array}{r} \begin{array}{l} 1,0094137 \\ -e = ,0004602 \\ \text{Divisor } 1,0090 \\ \text{Divisor } 1,00895 \\ \text{Divisor } 1,0089535 \end{array} \quad \begin{array}{l} ,000464340511 = D \\ 40360 \\ 607405 \\ 605370 \\ 20351100 (02 = e \\ 20179070 \\ 1720700 (01 = e \\ 1008953 \end{array} \end{array}$$

Here I desist forming a new Divisor.

$$\begin{array}{l} \text{Last } r = ,149 \\ + e = ,000460201 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Last } r = ,149 \\ + e = ,000460201 \end{array}} \right\} ,149460201 = a \quad \text{PROB.}$$

and their Equations,

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PROB. X.

THE Radius of a Circle, and Chord of an Arch being given, To find the Chord of one Ninth part of that Arch.

This Problem needs no Scheme, but a consideration of the Equation arising from that in Page 70, viz. Of the Trisection of an Angle, There it was found that

$$3 R R a - a a a = R R c.$$

And if it be made $R = c = 1$ in this Problem as it was in that, then it will be

$$3 a - a a a = 1$$

And here we are to find a third part of that Arch whereof a , is the Chord, for $\frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ In order thereunto,

let it be made $3 y - y y y = c$

And for the Chord sought, put a ; then will

$$3 a - a a a = y, \text{ and } 9 a - 3 a a a = 3 y$$

Also, $27 a a a - 27 a a a a a + 9 a^7 - a^9 = y y y$.

And from these will arise this Equation,

$$a^2 - 9 a^7 + 27 a^3 - 30 a^3 + 9 a = 3 y - y y y = c$$

If as before, $c = 1$ the Chord of 60 Degrees, then will a , be the Chord of 6 Degrees, 40 Minutes.

Put $r + e = a$, then there will arise these, viz.

$$+ r^2 + 9 r^8 e + 36 r^7 e e = + a^2$$

$$- 9 r^7 - 63 r^6 e - 189 r^5 e e = - 9 a^7$$

$$+ 27 r^5 + 135 r^4 e + 270 r^3 e e = 27 a^3$$

$$- 30 r^3 - 90 r^2 e - 90 r e e = - 30 a^3$$

$$+ 9 r + 9 e \dots \dots \dots = 9 a$$

$c = 1$ then $r > 1$ but $r < 2$ because we are to find the Ninth part of the Arch; Let $r = 1$ which being ordered as the Species directs, will produce these Numbers.

$$+ ,000000001 + ,00000009e + ,0000036ee$$

$$- ,0000009 - ,000063e - ,00189ee$$

$$+ ,00027 + ,0135e + ,27ee$$

$$- ,03 - ,9e - 9,0ee$$

$$+ ,9 + 9,0e$$

Thar

82 Resolving of Problems,

That is $870269101 + 8,11343709e - 8,7318864ee = 1$
 Then $8,11343709e - 8,7318864ee = ,129730899$
 And $,92918e - ee = ,014857 = D$

Consequently, $\left\{ \frac{D}{,929 - e} = e \right.$

$$\begin{array}{r} -e = ,92916 \\ \quad ,0163 \\ \hline \text{Divisor } ,91 \\ \text{Divisor } ,913 \\ \text{Divisor } ,9128 \end{array} \quad \begin{array}{r} ,014857 = D \\ 91 \\ \hline 5757 \\ 5478 \\ \hline 27900 \\ 27384 \end{array} \quad (,0163 = e$$

Last $r = ,1 + ,0163 = ,1163 = r$ I take $r = ,116$
 Which being ordered as before, will be

$$\begin{array}{r} +,000000003829 + ,0000000295e + ,00001017ee \\ - ,000002543598 - ,00015349e - ,003961ee \\ + ,000567092247 + ,0244430e + ,421419ee \\ - ,04682688 - 1,21104e - 10,44ee \\ + 1,044 + 9,0e \end{array}$$

$+ ,997737671478 + 7,813250428e - 10,0225433ee = 1$
 That is, $7,81325e - 10,0225433ee = ,002262328522$
 And then, $7795676e - ee = ,000225723995 = D$

Consequently, $\left\{ \frac{D}{,7795676 - e} = e \right.$

$$\begin{array}{r} -e = ,7795676) ,000225723995 = D \\ \quad ,0002896 \\ \hline \text{Divisor } ,7793 \\ \text{Divisor } ,77928 \\ \text{Divisor } ,7792780 \end{array} \quad \begin{array}{r} 15586 \\ 698639 \\ 623424 \\ 7521595 \\ 7013502 \\ 50809300 \\ 46756680 \\ 40526200 \\ 38963900 \\ 15623000 \\ 15585560 \end{array} \quad \begin{array}{l} (,0002 = e \\ (8 = e \\ (9 = e \\ (652 = e \end{array}$$

The Divisor remains
 without new forming.

$$\begin{array}{r} \text{Last } r = ,116 \\ + e = ,000289652 \\ e = ,116289652 \\ e = ,11629349,45e. \end{array}$$

I have chosen to prosecute these Angular Sections to this height (which is further than I ever saw) for that I was willing to shew by Example, with what ease and expedition this Method resolveth high Adfectèd *Æ*quations; the Root Converging as quick in them, as it doth in those of a lower Rank.

Also because from hence may be proposèd a much speedier, and easier way of approximation, towards the finding out the Circumference of a Circle, than what the Ancients had; for *Archimedes* (and others since him) proceeded therein, by the continual Bisection of an Arch, (as in Page 74 of this Tract) beginning with the Chord (or Subtence) of 60 Degrees; that is, of $\frac{1}{2}$ part of the Circumference, and from thence found the Chord of $\frac{1}{4}$, then of $\frac{1}{8}$, then of $\frac{1}{16}$, then of $\frac{1}{32}$, then of $\frac{1}{64}$, then of $\frac{1}{128}$, and so on, according to the nearness they intended to bring the Chord to the Arch it self. And this way of proceeding (as they resolvèd that *Æ*quation) requires Twenty Seven several Extractions of the Square Root (that is, Root out of Root) to obtain the Chord of $\frac{1}{128}$ part of the Circumference. And no doubt but they made use of those Numerous Extractions, for want of other Expedients to facilitate the Work; otherwise (it is but reasonable to suppose) they would not have spent so much time and labour, as must necessarily be required in the performance thereof; both which are by this Method much abreviated, and not only so, but may be more accurately performed, because here is not required the Extraction of a Surd Root, out of a Surd Root, so often repeated; for by the Resolving of one *Æ*quation, here is produced the Chord of $\frac{1}{4}$ part of the Circumference, beginning at the Chord of $\frac{1}{2}$, for $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; and by a Solution of the same *Æ*quation a Second time, it will produce the Chord of $\frac{1}{8}$ part of the Circumference; for $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$. And if further Exactness be required, it is but repeating it once more, and then it will produce the Chord of $\frac{1}{16}$ part of the Circumference.

Which

84 Resolving of Problems,

Which is nearer than can be obtained by the repetition of Fifty Three several Extractions, by the Method of Bisection before mentioned.

And the better to shew how near Two Solutions will approach to the (common received) Circumference of a Circle, whose Diameter is 2, I have inserted a Second Proceſs, taking the ſame Equation equal to the Chord laſt found, to wit,

$$a^3 - 9a^7 + 27a^5 - 30a^3 + 9a = ,116289652$$

And for the ſame Reason that the firſt $r = ,1$ in the laſt Proceſs, in this it muſt be made $r = ,01$

And here it is to be noted, that becauſe $r = ,01$ then will $r^9 - 9r^7$ the Two firſt Members of the Solution, have one 17, and the other 13 Cyphers before their ſignificant Figures; both which being of ſo ſmall value, may be rejected; by which the Solution will become contracted to this following.

$$\begin{array}{r} + 27r^5 + 135r^4e + 270r^3ee \\ - 30r^3 - 90r^2e - 90ree \\ + 9r + 9e \end{array} \} = ,116289652$$

$$\text{Then } \left\{ \begin{array}{l} + ,0000000027 + ,00000135e + ,00027ee \\ - ,00003 - ,009e - ,9ee \\ + ,09 + 9,000e \end{array} \right.$$

$$\text{Viz } ,0899700027 + 8,99100135e - ,89973ee = ,116289652$$

$$\text{Then } 8,99100135e - ,89973ee = ,0263196493$$

$$\text{And } 9,993e - ee = ,02925283,06 = D$$

$$\text{Conſequently } \left\{ \frac{D}{9,993 - e} = e \right.$$

$$\begin{array}{r} - e = 9,993) \quad ,02925283,06 = D \\ \quad ,002 \quad \quad 19982 \quad \quad (,002928 = e \\ \text{Diviſor } 9,991 \quad \quad 927083 \\ - e = ,0009 \quad \quad 899109 \\ \text{Diviſor } 9,9901 \quad \quad 2797406 \\ - e = ,00002 \quad \quad 1998016 \\ \text{Diviſor } 9,99008 \quad \quad 7993900 \\ \quad \quad \quad 7992064 \end{array}$$

$$\text{Laſt } r = ,01 \\ + e = ,002928 \} = ,012928 = x.$$

But

Concerning the Circle. 85

But if more Exactness be required, it may then be called a New r , for a Second Operation, (as before) which for brevities sake I shall omit, and rest satisfied with this one, as being sufficient to shew how near, and with what ease this Approach hath been made.

This Chord $,012928 = a$, is near the $\frac{1}{486}$ part of the Circumference, or Periphery of that Circle, whose Diameter is 2, though less than Just (it being only the side of the inscrib'd Polygone) and consequently if it be Multiplied into the Denominator of that Fraction, the Product will be near the length of the whole Periphery,

That is, $,012928 \times 486 = 6,283008$ the Periphery which is something less than that generally received, to wit, $6,283185$ as may be seen in the works of every Geometer; some of which are pleased to call us of *Metius*, *Van Culen*, *Snellius*, or *Hugenius*, &c. as Discoverers of this Number. But as to the Reason thereof, or by what Methods it was discovered, therein they are silent, and leave the Reader to exercise an Implicite Faith; or be at the trouble of searching into the Volumes of those Authors for his satisfaction. But herein I have sav'd my Reader that Trouble, and (as I humbly conceive) given all, who please to consider thereof, ample satisfaction, how they may with a very little labour, approach as near the Truth as can be assigned (for an absolute Determination thereof cannot be obtained.) Other Methods of Approachment there are; but this by the Chord of an Arch (be it perform'd by what Section you please) I take to be the most Natural and Demonstrable of all others.

Now by the help of this Periphery, $6,283008$ (or if you please, that before mentioned $6,283185$) and its Diameter, not only any other Periphery, but also the Area of any Circle may be easily found, I shall pass over those Proportions that arise from Squaring the Periphery, or Multiplying half the Periphery, into half the Diameter, &c. usually laid down in most Authors, for finding the Area, And effect the same by a Proportion very obvious from

86 Of finding the Periphery,

the Annexed Diagram. First then for obtaining the Periphery of any Circle.

Suppose the Side of the Square, $AB = 2 = D$, the Diameter of the greatest inscribed Circle, then is $6,283008 = P$ the Periphery of that Circle (as before.)

Again, Suppose the Side of the included Square, $ab = 1 = d$, the Diameter of its inscribed Circle; Let $p =$ the Periphery thereof.

Then it will be

$$D : P :: d : p$$

That is,

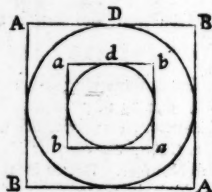
$$2 : 6,283 :: 1 : 3,1415$$

Consequently,

$$d : p :: D : P$$

That is,

$$1 : 3,1415 :: D : P$$



Here D is taken for any given Diameter whatsoever: Hence it follows, that if the Diameter of any Circle be Multiplied into $3,1415$ (or rather $3,1416$) the Product will be the Periphery of that Circle.

Next, for finding the Area; it's very obvious, that *As the Perimeter of the Square, Is to its contained Superficies or Area, So is the Periphery of the greatest inscribed Circle, To its Area.* (For a Demonstration of this Proportion, consider the Scheme, as if dissected into parts, which will then be very obvious.)

That is, $4D : DD :: P : A =$ the Circles Area.

Or, $4d : dd :: p : a =$ its Circles Area.

In Numbers, $4 : 1 :: 3,1416 : ,7854 =$ Area.

It was before found that $d : p :: D : P$

Therefore, $4d : p :: 4D : P$ But $4d : p :: dd : a$

Consequently, $dd : a :: DD : A$

That is, $1 : ,7854 :: DD : A =$ Area.

Here

Here DD is taken for the Square of any given Diameter; from hence it follows, that if the Square of any Diameter be Multiplied into $,7854$, the Product will be the Area of that Circle, in Square Measures of the same Name with the Diameter; viz. If the Diameter be Inches, then will the Area be square Inches, &c.

And what is here done, as to the Proportions betwixt the Circle and the Square, the same may be applied to the Ellipsis, and Parallelogram made of the Transverse and conjugate Diameters of that Ellipsis. But more of that (and other Conical Sections, with their Solids, &c.) hereafter. For if God permit, and I find due encouragement thereunto, I intend to present (my *Quandam* Brethren) the *Gaugers of Excise*, with a small Tract of *Gauging*, not only of Theorems or Rules, but also a Demonstration of each part thereof, both Algebraically, and in Words at length, with some Improvements therein not yet published. But leaving this Degression, let us proceed to some farther Application of what hath been already found.

From this Periphery $6,283008$ of the Circle, whose Radius is Unity, may be easily raised a Table or Canon of Natural Sines, Tangents and Secants. In order thereto, I take it for granted, that the Sine of one Minute doth so insensibly differ from the length of the Arch of one Minute, that without Error it may be taken from the same. Then it follows, that *As the Periphery in Minutes; Is to the Periphery in Equal parts of the Radius :: So is one Minute : To the parts agreeing to that one Minute.*

That is, $21600' : 6,283008 :: 1' : ,00029088$ = the Sine of one Minute, which agreeth with the largest Tables I ever yet saw. Having got the Sine of one Minute, the Co-sine thereof is thus obtained, by the Second part of the following Scheme.

$SD =$ Radius, $DF =$ the Sine of the Arch DA ,
Then $SF = RD$ is the Co-sine of the said Arch.

88 To make a Table of Sines.

But the $\square SD - \square DF = \square SF$.

Therefore $\sqrt{\square SD - \square DF} = SF$.

That is, From the Square of the Radius, Subtract the Square of the Sine of $1'$, the Square Root of the Remainder will be the Co-sine of $1'$.

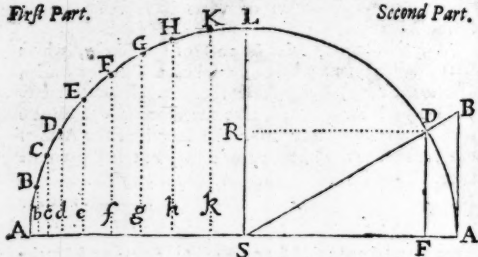
In Numbers $1 - .000000084612 = .999999915388$
The Square Root thereof is $.99999995 =$ the Co-sine required.

The Sine and Co-sine of one Minute, being thus found, all the rest of the Sines in the Quadrant may be easily rais'd, by Mr. Michael Davies's Sinical Proportion.

Which (if I understand aright) may be thus propos'd : If a Rank of Arches be equally different, such as we suppose the Arches $AB, BC, CD, DE, EF, \&c.$ in the Quadrant ASL , of the First part of the Schem,

First Part.

Second Part.



whose Sines are b, c, d, e, f, g, h, k , and $R =$ Radius, or Sine of 90 Degrees.

Then the Proportions will run thus,

$$c : b + d :: d : c + e, \text{ or } c : b + d :: g : f + h$$

Again, $f : e + g :: k : h + R, \text{ or } d : c + e :: e : b + h \&c.$

That

To make a Table of Sines. 89

That is, $\left\{ \begin{array}{l} \text{As any Sine in the Rank : Is to the Sum of the} \\ \text{Two Sines next it on each side :: So is any other} \\ \text{Sine in the Rank : To the Sum of the Two} \\ \text{Sines next it on each side, or to the Sum of} \\ \text{any Two Sines alike distant on each side it.} \end{array} \right.$

From hence it follows, that if the Radius be made the Middle Term of Two Arches, then the Two Sines on each side it, may be made the Sines of Two Arches, one of them Less than a Quadrant, by one Minute, and the other of them greater than a Quadrant by one Minute; the same Sine being common to both Arches.

Then it will be $R : 2k :: b : c + 0 :: c : b + d. \&c.$

That is, $\left\{ \begin{array}{l} \text{As the Radius : Is to the double Co-sine of one} \\ \text{Minute :: So is the Sine of one Minute, To the} \\ \text{Sine of Two Minutes, and of } 0' :: \text{And so is} \\ \text{the Sine of } 2' : \text{To the Sum of the Sines of } 3' \\ \text{and } 1' :: \text{And so is the Sine of } 3' : \text{To the Sum} \\ \text{of the Sines of } 4' \text{ and } 2'. \text{ And so on in a suc-} \\ \text{cessive order.} \end{array} \right.$

Now if from the Sum of the Sines of $3'$ and $1'$, be taken the Sine of $1'$, the Remainder will be the Sine of $3'$. And the like, if the Sine of $2'$, be taken from that of $4' + 2'$, the Remainder will be the Sine of $4'$, &c.

And by this Method of proceeding, all the Sines in the Quadrant may be easily calculated, by *Addition* and *Subtraction*. For the Radius, or first Term in the Proportions being Unity, *Division* is escaped; and because the Second Term varies not, if a Tariffa, or small Table be made thereof to all the Nine Digits, then *Multiplication* is also escaped; for by the help of that Tariffa, the whole Work may be performed by *Addition* and *Subtraction* only.

Having thus made the Sines, the Tangents and Secants are easily found by the following Proportions. See the Second part of the foregoing Scheme.

90 To make a Table of Tangents.

$SD = SA$ the Radius, $DF =$ the Sine of the Arch DA .

$SF =$ the Co-sine of that Arch (as before.)

$BA =$ the Tangent, and SB the Secant of the same Arch.

$$\text{Then } \begin{cases} SF : DF :: SA : BA \\ SF : SD :: SA : SB \end{cases}$$

That is, $\left\{ \begin{array}{l} \text{As the Co-sine of any Arch, is to the Sine of} \\ \text{that Arch; So is the Radius, To the Tangent} \\ \text{of the same Arch.} \end{array} \right.$

Again, $\left\{ \begin{array}{l} \text{As the Co-sine of any Arch, Is to the Radius;} \\ \text{So is the Radius, To the Secant of that Arch.} \end{array} \right.$

I omit inserting the Arithmetical Operations suitable to these several Proportions, they being so obvious in themselves, there needs no Example to explain them.

Other Varieties of Proportions there are, betwixt the Sines, Tangents, Secants, and their Complements, which I forbear laying down at this time, these being sufficient to accomplish what I here design'd; which is only to shew how the Canon of Natural Sines and Tangents, &c. may be raised (or made) from the effects of the Tenth Problem.

Having once made the Canon of Natural Sines and Tangents, &c. it will not be difficult to compose the Artificial; they being only the Logarithms of the Natural; respect had to the Characteristick of the Radius, which for Conveniency (not necessity) is always made 10, as tho the Radius of the Natural were 10000000000. Notwithstanding it may be more or less at pleasure; the Characteristick of the Radius being thus assign'd, that of the other Sines will be 9, till it come to the Sine of 5 Degr. 44', and there it changeth to 8. The Reason is plain, from the Decrease of one Place in the Natural Sines.

But

Making of Logarithms. 91

But for a further help herein, (and to gratify the Young Algebrist) I have here inserted a short Specimen of the Nature and Construction of Logarithms, which were first invented by the Lord *Nepeir*, Baron of *Merchiston* in *Scotland*; who ingeniously contrived how to perform *Multiplication* or *Division* of Natural Numbers; by Adding or Subtracting certain Artificial Numbers (fitted to correspond with the Natural,) called Logarithms. This Invention of his (no doubt) arose from a mature consideration of the Mutual Agreement that is found betwixt Numbers in a Geometrical Progression, and those in Arithmetical Progression.

For Instance, Suppose a Rank of Numbers in a Geometrical Progression; and to them let there be assigned a Rank of Corresponding Numbers in Arithmetical Progression.

Viz. $\begin{cases} 1. 2. 4. 8. 16. 32. 64. 128 \text{ \&c. Geometrical} \\ 0. 1. 2. 3. 4. 5. 6. 7 \text{ \&c. Arithmetical} \end{cases}$

It is very perceptible, That as the Numbers in the Geometrical Progression are produced by *Multiplication* or *Division*, those in the Arithmetical Progression are produced by *Addition*, or *Subtraction*; as doth appear by this Example.

Viz. $\begin{cases} 4 \times 32 = 128 \\ 2 + 5 = 7 \end{cases} \text{ or } \begin{cases} 128 \div 32 = 4 \\ 7 - 5 = 2 \end{cases} \begin{matrix} \text{Geometr.} \\ \text{Arithmet.} \end{matrix}$

Again, $\begin{cases} 1. 10. 100. 1000. 10000. 100000 \text{ \&c. Geometric.} \\ 0. 1. 2. 3. 4. 5 \text{ \&c. Arithmetical.} \end{cases}$

The same Agreement is betwixt these latter, as was between the Two First Ranks.

Viz. $\begin{cases} 1000 \times 10 = 10000 \\ 3 + 1 = 4 \end{cases} \text{ or } \begin{cases} 100000 \div 1000 = 100 \\ 5 - 3 = 2 \end{cases} \begin{matrix} \text{Geom.} \\ \text{Arith.} \end{matrix}$

Either of these Examples do sufficiently shew the Reason, and very Ground of Logarithms.

And

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And from the latter of these it was, that the Prime Logarithms, or Characteristicks, were first assigned, as appears by this Table.

<i>Natural Num.</i>	<i>Logarithms.</i>
1	0,0000000
10	1,0000000
100	2,0000000
1000	3,0000000
10000	4,0000000
100000	5,0000000

Having laid this Foundation, the next work was to find out the Logarithms of the intermediate Numbers situated betwixt 1 and 10, viz. of 2, 3, 4, 5, 6, 7, &c. And of those betwixt 10 and 100, viz. of 11, 12, 13, 14, 15, &c. and so on for the rest. This was a Work of some Difficulty, and very Laborious.

The First Step in order thereunto (as I conceive) was to find out a Rank of continual Means betwixt 10 and 1, so as that the last (and least thereof) might be a mix'd Number less than 2, and so near 1, as to have such a Number of Cyphers before the Significant Figures thereof, as was intended the places of Logarithms in the Table should consist of. Which Means are to be found, by Extracting the Square Root of 10 (having first annexed a Competent number of Cyphers thereunto.) Then Extracting the Root of that Root, and so by a continued Extraction of Root out of Root, until there be a Root so qualified as before mentioned. Which to make a Table to Seven Places in the Logarithms, will require Twenty Five several Extractions, the last of which will produce this Number, 1,00000006862238

The next Step, was to find out a Number betwixt (1) and (0) in Arithmetical Progreſſion, that might truly correspond with the Mean before found (betwixt 10 and 1,) such a Number must consequently be its Logarithm. And this

Making of Logarithms. 93

this may be found by a continual bisection (or halving) of 1, so often as was the Number of the foregoing Extractions (to wit, Twenty Five,) the last of which Bisections will produce 0,000000029802322 &c. the true Logarithm of 1,00000006862238.

For as 1,00000006862238 by Twenty Five continued Involutions (*viz.* first into it self, then that Product into it self, and so on successively) will produce 10 so will 0,00000002980232 by the like Number of doublings and redoublings, produce 1.

This Mean (or Number) and its Logarithm being thus found, it will follow by Proportion.

As the Significant Figures of this Mean : Is to the Significant Figures of its Logarithm : So is the Significant Figures of any Mean, betwixt any given Number and 1 : (having Seven Cyphers before such Figures : as this hath) To the significant Figures of its Logarithm. To which must be prefixed Seven Cyphers to compleat it. After which being doubled, and redoubled according to the number of Extractions required to produce it's corresponding Mean, will at last discover the true Logarithm of the given Number. For the clearing of this, take an Example.

Suppose it were required to find the Logarithm of the Number 2, to Seven Places. First by a continued Extraction of Root out of Root, beginning at 2, find such a Mean (or Root as before) betwixt 2 and 1, as will have Seven Cyphers before it's Significant Figures ; which after Twenty Three several Extractions, will be this Number 1,00000008262958. Then according to the foregoing Proportions, it will be

$$6862238 : 2980232 :: 8262958 : 3588557$$

to which prefix Seven Cyphers, as before directed, then will 1,00000008269958 have for its Logarithm ,00000003588557 which being doubled and redoubled (as above said) will produce 0,30102997958658 the true Logarithm of 2, which being contracted to Seven Places, according to the first Design (and agreeable to the Seven

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Seven Places of Cyphers,) then it will become 0,3010299. But in all the Tables that I have seen, the Logarithm of 2 is 0,3010300. I conceive the Reason is, because the remaining Figures 7958658 come so near Unity of the last place in the retained Figures.

And by the same Method that this Logarithm of 2 is made, may the Logarithm of any other Number be found. But when once the Logarithms of a few of the Prime Numbers, viz. of 3 . 7 . 11 . 13 . &c. (that is, of such Numbers as cannot be produced by the Multiplying of Two Integer Factors) are obtained, the rest may be easily compos'd by *Addition* and *Subtraction* only.

For as $3 \times 2 = 6$ So Log. of 3 + Log. of 2 = Log. of 6.

And as $10 \div 2 = 5$ So Log. of 10 - Log. of 2 = Log. of 5.

The like of all Numbers that have Aliquot Parts (that is, such Integer Numbers as may be divided by Integers.) And indeed the Logarithms of several of the Prime Numbers, may also be obtained by *Addition* or *Subtraction*, as might easily be shew'd, and is not difficult to conceive by any one, who but duly considers the Nature and Design of Logarithms, and hath been but a little conversant in their use, of which I shall forbear saying any thing at this time, and keep to my first Design herein, which was to give a brief Account of the Ingenious Author's Method (as I conceive it) of making the same (who undoubtedly found it a very difficult Work, by reason there is required so many several Extractions of Roots out of Roots, which must needs render it both Troublefome and Laborious.) Then to propose a different Method of raising the Logarithms of such Prime Numbers before mentioned (which require the Extraction of Roots to obtain their respective Means) with one Tenth part of the Trouble and Time required by the foregoing Method. And not only so, but more exact; for by our present Methods of Converging Series, the Root of any Power how high soever it be, is easily found at one single Extraction; and thereby the Errors which would arise by
Extracting

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Extracting a *Surd Root* out of a *Surd Root* (especially when often repeated) are avoided; and consequently such a Mean as may be required betwixt any Number, and Unity is thereby more exactly found.

Now how this may be performed, I here intend to shew, as briefly as I can. In order thereunto, take this as a Model.

Let a = the Root; or Mean required betwixt any Number and Unity,

$$\text{Then } \begin{cases} a^2 = \square a & . & a^4 = \square a^2 & . & a^8 = \square a^4 \\ a^{16} = \square a^8 & . & a^{32} = \square a^{16} & . & a^{64} = \square a^{32} \\ a^{128} = \square a^{64} & . & a^{256} = \square a^{128} & . & a^{512} = \square a^{256} \end{cases}$$

And so on successively with the Indices in Geometrical Progression, until the power of (a) be made equal to such a Term in that Progression, as that the Root, or value of (a) may have betwixt Unity, and its significant Figures, so many Cyphers, as are the intended Number of Places in the Logarithms.

For instance, let it be required to find the Mean between 10 and 1 (as in Page 92) then the Power of (a) must be $a^{33554432} = 10$ this Index 33554432 being the 25th Term in Geometrical Progression, which may be thus determined.

Let 1 the Characteristick, or Logarithm of 10, be divided by such a Term in Geometrical Progression, as will cause such a Number of Cyphers to be before the Significant Figures in the Quotient, as are required to be before the Figures of the Root (a) suppose 7 (as before)

$$\text{Then } 1 \div 33554432 = .00000002980232 \text{ \& c.}$$

which is the true Arithmetical Mean (as before found, by a continual Bisection of 1) correspondent to that signified by (a) And therefore the value of (a) found by extracting the respective Root of 10 = $a^{33554432}$ will be the Mean required,

Viz. 1.00000006862238 whose Log. is .00000002980232 These being found, are the Foundation of the rest, as before.

Then

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Then suppose it be required to find the Logarithm of any of the Prime Numbers; if you please that of 2. In order thereunto, Let a = the Root, or Mean sought betwixt 2 and 1 (as before) then must (a) be continually Involved, (as by the above Model) until its Index be equal to the greatest Term in Geometrical Progression, whose Number of Places of Figures are to be equal to the Number of required Cyphers before (a) to wit 7. According to which the Power of (a) will be $a^{8388608} = 2$ (this 8388608 being the 23d Term in Geometrical Progression) consequently the respective Root of $2 = a^{8388608}$ will be the Mean required.

Example.

$$\text{Let } r + e = a$$

$$\text{Then will } r^{8388608} + 8388608 r^{8388607} e + 35184367894528 r^{8388606} ee = a^{8388608} = 2$$

$$\text{Suppose } r = 1$$

$$\text{Then } 1 + 8388608e + 35184367894528ee = 2$$

$$\text{That is } 8388608e + 35184367894528ee = 1$$

Each part being divided by the Coefficient found prefixed to ee , viz. 351843 &c. Then it will become

$$,00000023e + ee = ,00000000000000284 = D$$

$$\text{Consequently, } \left\{ \frac{D}{,00000023 + e} = e \right.$$

$$\begin{array}{r} ,00000000000000284 = D \\ +e = ,000000008 \\ \hline \text{Divisor } ,00000031 \end{array} \quad \begin{array}{r} 248 \\ 36 \end{array} \quad (,00000008 = e$$

$$\begin{array}{l} \text{First } r = 1, \\ + e = ,00000008 \\ \text{New } r = 1,00000008 \end{array}$$

Which being duly Involved (in the same order as the Model denotes) and Multiplied into the respective Coefficients, will

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will then produce these Numbers

Viz. $1,9563638967 + 16411168e + 68833416066289ee = 2$

Then $16411168e + 68833416066289ee = ,0436361033$

And $,0000002384e + ee = ,00000000000000063393 = D$

$$\text{Consequently, } \left\{ \frac{D}{,0000002384 + e} = e \right.$$

$$,0000000000000000063393 = D$$

$$+ e = ,0000002384 \quad \begin{array}{r} 480 \\ 15393 \end{array} \quad (,00000000263 = e$$

$$\text{Divisor } ,0000000240 \quad \begin{array}{r} 14460 \\ 9330 \end{array}$$

$$\text{Divisor } ,0000002410 \quad \begin{array}{r} 7230 \end{array}$$

$$\text{Last } r = 1,000000008$$

$$+ e = ,00000000263$$

$$\text{New } r = 1,000000008263$$

I take only $1,0000000286 = r$. The which being Involved, and ordered as before, will produce these following Numbers, viz.

$$1,999503684867 + 16773028e + 70351267454084ee = 2$$

$$\text{Then } 16773028e + 70351267454084ee = ,000496315133$$

$$,0000002384186e + ee = 00000000000000000705481443 = D$$

$$\text{Consequently, } \left\{ \frac{D}{,0000002384186 + e} = e \right.$$

$$,00000000000000000705481443 = D$$

$$+ e = ,0000002384186 \quad \begin{array}{r} 47686 \\ 2286214 \end{array} \quad (,0000000000295 = e$$

$$\text{Divisor } ,00000023843 \quad \begin{array}{r} 2146023 \end{array}$$

$$\text{Divisor } ,000000238447 \quad \begin{array}{r} 14019143 \end{array}$$

$$\text{Divisor } ,0000002384481 * \quad \begin{array}{r} 11922405 \end{array}$$

$$\begin{array}{r} * 20967380 \\ 19075848 \end{array} \quad (879 = e$$

* Here I desist forming a new Divisor, and make use of the Abridgment, as in Pag. 52, and elsewhere.

$$\begin{array}{r} 1891532 \\ 1669136 \\ 222396 \\ 214596 \end{array}$$

$$\text{Last } r = 1,0000000826$$

$$+ e = ,0000000000295879$$

$$s = 1,0000000826295879$$

K

The

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This value of $a = 1,0000000826295879$ is the Geometrical Mean betwixt 2 and 1, as was required; (agreeable to that before found, by Twenty Three several Extractions.) And by this Method of proceeding, may be found the Mean betwixt 10 and 1 viz. $1,00000006862238$ or betwixt any other of the (before mentioned) Prime Numbers and Unity, as might easily be shewed. But for brevities sake, I shall omit giving more Examples thereof, this one being sufficient (especially to the Ingenious) if well considered, and but once understood, to shew the nature of, and manner how; to proceed upon the like occasion, of finding any proposed Mean. The next thing will be to find the Logarithm of the Number, from whence such Mean was produc'd; which may be thus perform'd.

First, find its corresponding Arithmetical Mean, or Logarithm, by Proportion (as in Page 93.) Then Multiply that corresponding Mean (so found) into the Index Number of such Power as the Geometrical Mean was produced from; that Product will be the Logarithm of the given Number (without a continued doubling and redoubling as before.) For the clearing of this, let it be required to compleat the Logarithm of 2.

Having first found $1,00000006862238$ the proper Geometrical Mean betwixt 10 and 1 Also its corresponding Logarithm, $0,00000002980232$ (as before directed) with them, and the Mean betwixt 2 and 1, last found, viz. $1,0000000826295879$ make use of the above-mentioned Proportion (as in Page 93) viz.

$$6862238 : 2980232 :: 826295879 : 358855729$$

to which prefix Seven Cyphers to compleat it (as before) Then it will become $0,0000000358855729$ This Number being Multiplied into the power of a (what that is, see Page 96) will produce the Logarithm of 2.

$$\text{Viz. } 0,0000000358855729 \times 8388608 = 0,30103000391352$$

But according to the first Design, it is required to have but Seven Places, viz. $0,3010300$ which is the true Logarithm of 2 without any defect

Thus

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Thus I have presented you with a New and Expeditionary Method of making Logarithms, which if they were requir'd to 14 or 15 Places (I can Modestly say) they might then be made with one Twentieth part of the Time and Trouble required by the first Method.

Before I conclude, it will not be amiss to insert an Example or Two of such Equations as have all their Terms in them. Suppose this Equation were given,

$$aaa + baa - da = G, \text{ what's the value of } a?$$

$$\text{In Numbers } aaa + 438aa - 7825a = 98508430$$

$$b = 438 \quad d = 7825 \quad G = 98508430$$

$$\text{Let } r + e = a.$$

$$\text{Then } \begin{cases} rrr + 3rre + 3ree = aaa \\ brr + 2bre + bee = baa \\ -dr - de = -da \end{cases}$$

Suppose $r = 4$ then $rr = 16$ $rrr = 64$ $brr = 64$ and $dr = 0$ (for there is no Figure under the First Point at d) Then doth $rrr + brr = 128 > 98$ Ergo $r < 4$

$$\text{Let } r = 300 \text{ then it will be}$$

$$\begin{array}{r} + 27000000 + 2700000e + 90000e \\ + 39420000 + 2628000e + 43800e \\ - 2347500 - 7825e \end{array} \quad \left. \vphantom{\begin{array}{r} + 27000000 \\ + 39420000 \\ - 2347500 \end{array}} \right\} = G$$

$$\text{That is, } 64072500 + 5249750e + 133800e = G$$

$$\text{Then } 5249750e + 133800e = 34435930$$

$$\text{And } 3920 + 00 = 25736 = D.$$

$$\text{Consequently, } \left\{ \frac{D}{392 + e} = e \right.$$

$$\begin{array}{r} + e = \frac{392}{5} \quad 25736 = D \\ \text{Divisor } 44 \quad \underline{220} \\ + e = \frac{48}{8} \quad 3736 \\ \text{Divisor } 450 \quad \underline{3600} \end{array} \quad \begin{array}{l} (5 = e) \\ (8 = e) \end{array}$$

$$\text{Last } r = 300$$

$$+ e = 58$$

$$\text{New } r = 358 \text{ I take but } 350 = r$$

This New r being Involved and Multiplied into the Coefficients,

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Coefficients, as the Species above directs, it will be

$$\begin{aligned} &+ 42875000 + 367500e + 1050ee \\ &+ 53655000 + 306600e + 438ee \\ &- 2738750 - 7825e \end{aligned}$$

That is, $93791250 + 666275e + 1488ee = G$.

Then $666275e + 1488ee = 4717180$

And $447,76e + ee = 3170,1478 = D$.

Consequently, $\left\{ \frac{D}{447,76 + e} = e \right.$

$\begin{array}{r} 447,76 \\ + e = \underline{6,99} \\ \text{Divisor } 453, \\ \text{Divisor } 454,6 \\ \text{Divisor } 454,75 \end{array}$	$\begin{array}{r} 3170,1478 = D \\ \underline{2718} \\ 452,14 \\ \underline{409,14} \\ 430078 \\ \underline{409275} \\ 208030 \end{array}$
--	--

$$\begin{aligned} \text{Last } r &= 350, \\ + e &= \underline{6,994} \\ a &= 356,994 \end{aligned}$$

In this Proceſs, I humbly conceive, there is no difficulty appears; that, which ſeemeth like one, is the placing of the firſt value of $e = 5$. But if it be conſidered, that there muſt be Three Places of Integer Figures in the Root (as appears by the Points) and that this 5 is to be the Second Figure thereof, it will be eaſie to determine that its place muſt be under the Second Figure of the Diviſor, viz. 9.

This Conſideration will be very helpful in all Caſes where the Root conſiſteth of moſt Integers: As for Decimal Figures they place themſelves,

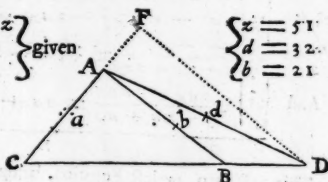
P R O B.

PROB. XII.

IN the Oblique Triangle, CAD , there is given the Side AD , and the Sum of the other Two Sides, viz. $CA + CD$; also within the Triangle, there is given the Line AB , so drawn, as to make the included Triangle CAB , Right-angled at A .

Thence to find out the Side CA .

Let
 $CA + CD = x$
 $AD = d$
 $AB = b$ } given
 $x = 51$
 $d = 32$
 $b = 21$



Draw the Line DF , parallel to AB , and continue the Side CA to F , then the Triangles, CAB , and CFD will be alike.

Let $CA = a$ $AF = e$ and $FD = y$.

Then,	1	$x - a = CD$, by the Prob.
1	2	$xx - 2xa + aa = \square CD$.
but	3	$xx - 2xa + aa = aa + 2ae + ee + yy$
3	4	$xx - 2xa = 2ae + ee + yy$
but	5	$dd = ee + yy$.
4	6	$xx - 2xa - dd = 2ae$
Let	7	$2f = xx - dd$
6, 7,	8	$2f - 2xa = 2ae$
8	9	$f - xa = ae$
10	10	$aa + bb = \square CB$, by the Scheme;
10	11	$\sqrt{aa + bb} = CB$.

K 3

But

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But	12	$CB:CA::CD:CE$
That is,	13	$\sqrt{aa+bb}:a::z-a:a+e$
13	14	$za-aa \div \sqrt{aa+bb} = a+e$
9	15	$f-za \div a = e$
15	16	$f-za+aa \div a = a+e$
14, 16,	17	$\frac{za-aa}{\sqrt{aa+bb}} = \frac{f-za+aa}{a}$
17	18	$\frac{zaa-aaa}{\sqrt{aa+bb}} = f-za+aa$
18	19	$za^3 - a^3 = f-za+a^2 \times \sqrt{aa+bb}$
Then	20	$\frac{za^3 - a^3}{f-za+aa} = \sqrt{aa+bb}$
And	21	$\square \frac{zaa-aaa}{f-za+aa} = aa+bb$

This 21st Step, or last Equation, being first Involved, then brought out of the Fraction, and reduced into Numbers, will produce this Equation.

$$-2018aaaa + 125609aaa - 2464230,25aa + 35468307a = 274183922,25$$

Each Member thereof being divided by 2018, the Factor of the highest Power of a , that so the same may be cleared, will bring down the Equation to this,

$$-aaaa + 62,1aaa - 1221,12aa + 17575,969a = 135869,1388$$

Let the Equation be made

$$-aaaa + baaa - daa + fa = G.$$

Here $62,1 = b$, $1221,12 = d$
and $17575,969 = f$, $135869,1388 = G.$

Put

Put $r + e = a$, then.

$$\begin{aligned} - rrrr - 4rrre - 6rree &= -a^4 \\ + brrr + 3brre + 3bree &= +ba^3 \\ - drr - 2dre - dee &= -daa \\ + fr + fe \dots \dots &= +fa \end{aligned}$$

Let $r = 10$, then it will be

$$\begin{aligned} - 10000 - 40000 - 60000 \\ + 62100 + 186300 + 186300 \\ - 122112 - 244220 - 122100 \\ + 175750 + 175750 \dots \dots \end{aligned} \Bigg\} = G.$$

That is, $105547 + 77830 + 4200 = 135869$

Then $77830 + 4200 = 30322$

And $185,60 + 00 = 721,95 = D.$

Consequently, $\left\{ \frac{D}{185,6 + e} = e \right.$

$$\begin{array}{r} 185,6 \quad 721,95 = D \\ + e = 3,8 \quad 564 \quad (3,8 = e \\ \hline \text{Divisor } 18,8 \quad 15795 \\ \text{Divisor } 189,4 \quad 15152 \end{array}$$

First $r = 10$,

$$+ e = 3,8$$

New $r = 13,8$

I take but Two Places, viz. $r = 13$, which being Involved, and duly Multiplied into the respective Coefficients, will produce these following Numbers.

$$\begin{aligned} - 28561,00 - 8788,000 - 1014,0000 \\ + 136433,70 + 31584,700 + 2421,9000 \\ - 206369,28 - 31749,120 - 1221,1200 \\ + 228487,60 + 17375,970 \end{aligned}$$

That is, $129991,02 + 8623,550 + 186,7800 = G$

Then $8623,550 + 186,7800 = 5878,1188$

And $46,1690 + 00 = 31,470814 = D.$

Con-

A SHORT
A P P E N D I X
 CONCERNING

Compound-Interest, and Annuities.

MY Design in this Appendix is only to shew how such Questions in Compound-Interest, &c. as have heretofore been very difficult, (if not impossible) to be performed (without the help of Logarithms,) may now be easily Resolved, by the foregoing Method of Extracting Roots, without the assistance of any Tables whatsoever.

In order thereunto, I shall first touch upon those General Heads that comprehend the whole: Then lay down such Theorems, as (if well considered, and truly applied) will Resolve all Questions therein, giving some few Examples in Numbers, especially in those Questions whose Solution depends upon the Extraction of Roots.

And herein I'll not dispute the Lawfulness or Unlawfulness of this sort of Interest; only thus far it seems (to me) rational, that if it be lawful to take Interest at all, then it's as lawful to put out the Interest-Money to use, as the Principal: If so, why may it not be joyned to the first Principal (or prime Stock) as it becomes due, thereby creating (as it were) a new Principal, which if granted, then is the growing Interest properly called Interest upon Interest, or Compound Interest; the Computation of which, is no more but to find out a true, or just Equation of Payments.

For

For instance, Suppose A and B , to represent Two Persons: Then if A pay to B any Sum of Money before the time it becomes due (or ought to be paid at) it is but reasonable that B should make a proportional Allowance by Discount to A , for such time (according to the Rate of Interest agreed upon.) But on the contrary, if A pay not such Sum to B , until after the time it became due, by the same Reason A should make an allowance to B , by Amount or Increase for the time he kept such Sum after it became due. Upon these Two Cases depends all Questions relating to Interest, either Simple or Compound.

For the Resolving of which, the best way is to state or form the Question proposed, as though the Demand lay upon one Pound only: And having found a fit Answer (according to the import of the Question) for one Pound, (to a convenient Number of Decimal Parts) Multiply the Sum, or Number of Pounds, &c. proposed in the Question, into that Answer agreeing to one Pound, the Product arising therefrom, will be the Answer required.

For the easier expressing of the several parts given or sought, I make use of these Letters.

P = the Principal } given or sought in any Question.
 t = the Time } or Number of Years or Days, &c.
 a = the Amount } given, or sought.
 s = the Sum } of one Pound, for one Year, or
 Day, &c. according to any Rate
 proposed.
 z = the Sum } or Amount of Principal and In-
 terest, given or sought.

Where Note, That in the Cases of Compound-Interest, (i) Is the Index of the Power of (s) .

Now for the raising a General Theorem, by which all Questions in Compound-Interest may be Resolved, consider these Two Proportions.

First, $1 : a :: a : aa :: aa : aaa :: aaa : aaaa$ &c. in &c.

That

That is, $\left\{ \begin{array}{l} \text{As one Pound : Is to its Amount, (or one Pound} \\ \text{with its Interest) at one Years End :: So is that} \\ \text{Account : To the Amount of one Pound at two} \\ \text{Years, and so on.} \end{array} \right.$

From hence it is evident, That Compound-Interest is grounded upon a Rank of Geometrical Proportionals Continued; the last of which is known by the Number signified by (t) and is a^t .

Secondly, $1l : a^t :: P : z$ Ergo $Pa^t = z$

That is, $\left\{ \begin{array}{l} \text{As one Pound : Is to the Account of one Pound, (An} \\ \text{year} \\ \text{for any time proposed :: So is 100 . 100 . 100 or} \\ \text{any Sum proposed : To its Amount, for the same} \\ \text{time.} \end{array} \right.$

From these Two Proportions, I presume the General Theorem $Pa^t = z$ is sufficiently demonstrated, and may be clearly understood.

Quest I. Suppose 250 l. hath been at Interest Seven Years, What doth it amount to, at 6 per Cent. per Annum, Compound-Interest?

Here is given $P = 250$ $t = 7$ and $a = 1,06$ For $100 : 6 :: 1 : 1,06 = a$ the First Year; Then it follows, that if a be Involved so often, until its Index $= t$, viz. $a^7 = a^t$. And then Multiplied into P it will produce z as appears by the Theorem $Pa^t = z$

But $a = 1,06$ \odot 7 times $= 1,50363$

And $250 \times 1,50363 = 375,9075 = z$

That is, 375 l. 18 s. 2 d. is the Sum produced from 250 l. having been at Compound-Interest 7 Years, (as above proposed.)

Quest II. Suppose 375 l. 18 s. 2 d. were to be paid Seven Years hence, What is it worth in ready Money, abating 6 per Cent. per Annum, Compound-Interest?

Here is given $z = 375,9075$ $t = 7$ and $a = 1,06$ to find P .

General Theorem $Pa^t = z$, therefore $z \div a^t = P$

But

But $a = 1,06$ \odot 7 times $= 1,50363$
 And $375,9075 \div 1,50363 = 250 = P$.
 Answer it is worth 250 l. Ready Money.

Quest. III. Suppose 250 l. hath been at Interest, and the Amount is 375 l. 18 s. 2 d. at 6 per Cent. Compound Interest, How long hath it been foreborn.

Here is given $P = 250$ $z = 375,9075$ And $a = 1,06$ for one Year. Thence to find $t =$ the Index Power of a .

General Theorem $Pa^t = z$. Therefore $z \div p = a^t$

Consequently if a^t be continually Divided by a , until it become $a \div a = 1$ the Number of such Divisions will be $= t$. For such Number of Divisions discovers how oft (a) was involved.

But $375,9075 \div 250 = 1,50363 = a^t$.

And $1,50363 \div 1,06 = 1,418518$

Again, $1,418518 \div 1,06 = 1,338225$

And so on, until it become $1,06 \div 1,06$ which will be at the Seventh Operation.

Then will $t = 7$ the Number of Years required.

Quest IV. Suppose 250 l. hath been foreborn Seven Years, and the Debtor is willing to give up both Principal and Interest, proffering 375 l. 18 s. 2 d. to be cleared, What Rate of Interest per Cent. (allowing Compound Interest) doth hereby offer to the Creditor?

Here is given $P = 250$ $z = 375,9075$ and $t = 7$
 Thence to find a

General Theorem $Pa^t = z$ Therefore $z \div p = a^t$

That is $a^t = a^7$ Consequently, $\sqrt[t]{z \div P} = a$

But $z \div P = 1,50363 = G$

Put $r + e = a$.

$$\begin{aligned} & r^7 + 7r^6e + 21r^5ee = a^7 = G \\ \text{Then } & \left\{ \begin{aligned} \frac{1}{7}r^7 + r^6e + 3r^5ee &= \frac{1}{7}G \\ \frac{1}{7}rr + re + 3ee &= \frac{1}{7}G \div r^5 \\ re + 3ee &= \frac{\frac{1}{7}G}{r^5} - \frac{1}{7}rr = D \end{aligned} \right. \end{aligned}$$

Hence

Hence ariseth this

$$\text{Theorem } \left\{ \frac{D}{r + 3e} = e \right.$$

Let $r = 1$

$$\begin{array}{rcl} 1,50363 & = & G \\ 3,214804 & = & \frac{1}{7} G \div r^5 \\ - 1,42857 & = & \frac{1}{7} rr \\ \hline 3,071947 & = & D \quad (06 = e) \\ 708 & & \end{array}$$

$$\begin{array}{rcl} r & = & 1, \\ \div 3e & = & ,18 \\ \text{Divisor} & & 1,18 \end{array}$$

$$\begin{array}{rcl} \text{First } r & = & 1, \\ + e & = & ,06 \\ \text{New } r & = & 1,06 \end{array} \quad \begin{array}{rcl} 3,21480428 & = & \frac{1}{7} G \\ 3,16051432 & = & \frac{1}{7} G \div r^5 \\ - 1,16051428 & = & \frac{1}{7} rr \\ \hline 3,00000004 & & \end{array}$$

From hence it appears, that $1,06 = a$

Then $1 : 1,06 :: 100 : 6 =$ the Rate of Interest required.

But if in any Questions, either of Interest or Annuities, the time given or sought, be not terminated by whole Years (as before) but lies upon Weeks, Months, Quarters, half Years, Three Quarters, &c. the best Method for Resolving such Questions, is first to Reduce such Broken or Fractional parts of the Year into Days, viz. $\frac{1}{2} = 7$ Days, $\frac{1}{4} = 30,4$ Days, $\frac{1}{8} = 91,25$ Days, $\frac{1}{16} = 182,5$ Days, $\frac{1}{32} = 273,75$ Days, and so for any odd Number of Days that falls betwixt such even parts of the Year. This being done, find an Answer, according to the Demand of the Question (and agreeing to one Pound as before) for the Number of Days proposed. Now for the performance of this (and as a Foundation for the answering all such Demands as lies upon the parts of a Year, be they Equal or Unequal) it will be requisite to Resolve this following Question.

What is the Amount (or if you please the Interest) of one Pound for one Day; at 6 per Cent. per Annum, Compound Interest?

For the Amount sought, put a , then (according to the last Line in Page 106) it will be,

$$1 : a :: a : aa :: aa : aaa :: aaa : aaaa \div \div$$

L

That

That is, $\left\{ \begin{array}{l} \text{As one Pound: Is to its Amount for one Day :: So} \\ \text{is that Amount : To the Amount for Two} \\ \text{Days :: And so is that of Two Days : To that} \\ \text{of Three Days : And so on to 365 Days.} \end{array} \right.$

The last of which will be $a^{365} = 1,06$

Put $r + e = a$

$$\begin{cases} r^{365} + 365r^{364}e + 66430r^{363}ee = 1,06 = G \\ \frac{1}{365}r^{365} + r^{364}e + 182r^{363}ee = \frac{1}{365}G \\ \frac{1}{365}rr + re + 182ee = \frac{1}{365}G \div r^{363} \\ re + 182ee = \frac{1}{365}G - \frac{1}{365}rr = D \end{cases}$$

$$\text{Theorem } \left\{ \frac{D}{r + 182e} = e \right.$$

$$\begin{array}{rcl} \text{Let } r=1, & ,00290410 = \frac{1}{365}G \div r^{363} \\ + 182e = ,0182 & - ,00273972 = \frac{1}{365}rr \\ \text{Divisor } 1,0182 & ,00016438 = D \\ & 10182 & (,00016=e \end{array}$$

$$\begin{array}{rcl} \text{New } r = 1,0001 & 625600 \\ + 182e = ,01092 & 606612 \\ \text{Divisor } 1,01102 & \end{array}$$

New $r = 1,00016$ for a Second Operation.

Then is $,00274025636372 = \frac{1}{365}G \div r^{363}$

And $-,00274060280986 = \frac{1}{365}rr$

Hence it appears the Excess lieth upon $\frac{1}{365}rr$, and therefore the Difference, Or New Resolvend, will have the Sign — and consequently it must be $-e$

$$\begin{array}{rcl} - ,00000034644614 = D \\ r=1,00016 & 30003162 (-,0000003=e) \\ - 182e = 1,0000546 & 46414520 (-464=e) \\ \text{Divisor } 1,0001054 & 40004216 \\ & 6410304 \\ & 6000630 \\ \text{Last } r = 1,00016 & 409674 \\ - e = 000000346.4 & 400040 \\ a = 1,0001596536 & \end{array}$$

This Value of a , is the Amount of 1 $l.$ for one Day; from

An Appendix.

III

from which, if 1 *l.* be Subtracted, the Remainder .0001596536 will be the Interest of 1 *l.* for one Day. Consequently, if any proposed Sum be Multiplied into either of these, the respective Product will be the Amount, or Interest of that Sum for one Day.

And if from hence, were Calculated a Table of the several Powers of *a*, it would then be

$$a . a^2 . a^3 . a^4 . a^5 . a^6 . a^7 = \text{the Amounts,}$$

$$1 . 2 . 3 . 4 . 5 . 6 . 7 = \text{the Days,}$$

and so continued on to $a^{365} = 1,06$ the Amount for 365 Days.

This Table would be of good use for the speedy Resolving of all Questions relating to Compound Interest, &c. for any number of Days less than one Year, as might be easily shew'd; but I shall omit giving Examples thereof, what hath been already deliver'd, being sufficient (if consider'd, and once understood) to render the whole Business of Compound Interest Easie: Neither indeed can that of *Annuities*, *Pensions*, and *Leases in Reversion*, &c. be then difficult; their difference being only this, Compound Interest is grounded upon a Rank of Geometrical Proportionals continually increasing (as in Page 107) the business of *Annuities*, &c. is grounded upon a Rank of Geometrical Proportionals (but) continually decreasing, And may be thus express'd.

$p =$ the { Pension, or Annuity; and is the first or greatest Term in the Progression.
 $t =$ the { Time of Continuance (as in Interest) and is the Number of all the Terms except the First.
 $a =$ the { Rate of Interest for one Pound (as before) and is the common Ratio of all the Terms.
 $z =$ the { Sum of all the Terms, except the first, and is the Price, or present worth of any Annuity, or Pension, &c.

These premised, the Progression will be

$$p : \frac{P}{a} :: \frac{P}{a} : \frac{P}{aa} :: \frac{P}{aa} : \frac{P}{aaa} :: \frac{P}{aaa} : \frac{P}{aaaa} :: \frac{P}{aaaa} : \frac{P}{aaaaa} \&c. \text{ in } \frac{z}{a}$$

An Appendix.

until it become $\frac{P}{a^t}$ That is, until the Index of the Power of a , be equal to the time of Continuance of the Annuity. In this Progression $\frac{P}{a^t}$ is the last Term.

Then $z - \frac{P}{a^t}$ will be the Sum of all the Antecedents.

And $z - \frac{P}{a}$ will be the Sum of all the Consequents.

Then $\left\{ \begin{array}{l} \text{As one of the Antecedents : Is to its Consequent ::} \\ \text{So is the Sum of the Antecedents : To the Sum of} \\ \text{the Consequents. (vide Page 25)} \end{array} \right.$

That is, $P : \frac{P}{a} :: z - \frac{P}{a^t} : z - \frac{P}{a}$

$$\text{Ergo } zP - \frac{PP}{a} = \frac{zP}{a} - \frac{PP}{a^t \times a}$$

That is, $za - p = z - \frac{p}{a^t}$ Or $za - z = p - \frac{p}{a^t}$

By this Equation may all the Cases in Annuities or Pensions, &c. (that are bounded by Time) be Resolved, by transposing and ordering the several parts thereof, according as the Nature and Demand of the Question requires.

For instance, Suppose the Yearly Pension or Lease, Years of Continuance, and Rate of Interest were each given, Thence to find the worth thereof in present Money.

Here is given P , t , and a , to find z .

The Equation $za - z = p - \frac{p}{a^t}$ therefore $z = \frac{p - \frac{p}{a^t}}{a - 1}$

The like for finding any of the other Parts, viz. P , t , or a . I conceive it needless to suit Arithmetical Examples thereunto; they being perform'd in all respects like unto those of Interest. I shall therefore leave them to the Exercise of the Practitioner. To whom I wish all good success.

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